Simulation of MOD1 (2)

- SQNRs for different frequencies:

![Graph showing SQNR vs Input Amplitude for low-frequency and high-frequency inputs. Theoretical curve indicates a linear relationship.]

Note: "Bad model!"
Simulation of MOD1 (3)

- In-band quantization noise power:

\[ V_{\text{ref}} = \pm 1 \]

- OSR = 32

- OSR = 64

\[ u = \frac{1}{2} \]

idle tones (limit cycles)

mean square of inband noise
MOD1 Under DC Excitation (1)

- Idle tones:

$$y(n) = y(n-1) + u - v(n-1)$$

$$v(n) = \text{sgn}(y(n)), \quad \text{sgn} 0 = 1$$

$$y(n) = y(n-1) + u - \text{sgn}(y(n-1))$$

- \( u = y(0) = \frac{1}{2} \): \( v_{neq} = 1 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y(n) )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>( -\frac{1}{2} )</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( v(n) )</td>
<td>1</td>
<td>1</td>
<td>( -1 )</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- For \( u = 0.01 \), tones at \( k \cdot f_s/200! \) \( k = 1, 2, \ldots \)
Inband Tone Generation

\[ u = \frac{1}{500} > 0 \]
MOD1 Under DC Excitation (2)

- Let $u = a/b$, $a$ and $b$ odd integers, and $0 < a < b$. Also, let $|y(0)| < 1$. Then, the output has a period $b$ samples. In each period, $v(n)$ will contain $(b+a)/2$ samples of $+1$, and $(b-a)/2$ samples of $-1$.
- If $a$ or $b$ is even, the period is $2b$, with $(a+b)$ $+1$s and $(b-a)$ $-1$s.
  
  See p. 19, $a=1, b=2$

- If $v(n)$ has a period $p$, with $m+1$s and $(p-m)-1$s, the average $\bar{v} = (2m - p)/p$. Hence, $u = \bar{v}$ is also rational. Thus, rational dc $u \Leftrightarrow$ periodic $v(n)$.
- Periodic $v(n)$: pattern noise, idle tone, limit cycle. Not instability!
- For $u = 1/100$, tones at $k \cdot f_s/200$, $k = 1, 2, \ldots$ some may be in the baseband. Often intolerable!
Stability of MOD1

- MOD1 always stable as long as $|u| \leq 1$, (and $|y(0)| \leq 2$):

  $y(n) = \underbrace{[y(n-1) - \text{sgn}(y(n-1))]}_{\| \| \leq 1} + u(n) \leq 2$

  \[ \|u\| \leq 1 \]

- If $u > 1$ (or $u < -1$), $v$ will always be $+1$ (or $-1$) $\Rightarrow y$ will increase (or decrease) indefinitely.

- If $|u(n)| \leq 1$ but $|y(0)| > 2$, then $|y(n)|$ will decrease to $< 2$. Output spectrum always a line spectrum for MOD1 with dc input (rational or not).
The Effects of Finite Op-Amp Gain (1)

- Degraded noise shaping:

\[ q_2(n) = q_2(n-1) + C_1 \left( u(n) - v(n-1) - \frac{q_2(n)}{C_2(A+1)} \right) \]

\[ \text{pole: } p = 1 - 1/A \]

\[ Y(z) = p \frac{zU(z) - V(z)}{z-p} \]

\[ \text{NTF}(z) = 1 - pz^{-1} \rightarrow 1 - p = 1/A \]
The Effects of Finite Op-Amp Gain (2)

- Dead zones:

  Ideally: \( y(1) - y(0) + u - \text{sgn}(y(0)) = u - 1 < 0 \) \( v = -1 \)
  \( y(2) = (u - 1) + u + 1 = 2u > 0 \) \( +1 \)
  \( y(3) = 2u + u - 1 = 3u - 1 < 0 \) \( -1 \)

  \( y(k) = \begin{cases} 
  ku - 1 & \text{if } k \text{ is odd} \\
  ku & \text{if } k \text{ is even}
\end{cases} \)

  for \( u > 0 \), eventually \( ku > 1 \) and two 1's occur.

  \( y(n) = p y(n-1) + u - \text{sgn}(y(n-1)) \), \( p = 1 - 1/A < 1 \)

  \( y(1) = y(0) + u - \text{sgn}(y(0)) = u - 1 < 0 \) \( v = -1 \)
  \( y(2) = pu - p + u + 1 = (1 + p)u + (1 - p) > 0 \) \( +1 \)
  \( y(3) = p(1 + p)u + p(1 - p) + u - 1 = (1 + p + p^2)u - (1 - p + p^2) < 0 \) \( -1 \)

  \[ y(k) = \sum_{i=0}^{k-1} p^i u + (-1)^k \sum_{i=0}^{k-1} (-p)^i \]

  For odd \( k \neq \) ? \( k \rightarrow \infty \), \( y(k) \leq 0 \)

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The Effects of Finite Op-Amp Gain (3)

• For $\bar{v} > 0$,
  (Two 1’s occuring)

\[
\frac{u}{1-p} > \frac{1}{1+p}
\]

\[
u > \frac{1-p}{1+p} = \frac{1/A}{2 - 1/A} \approx \frac{1}{2A}
\]

For $A \approx 10^3$:

Dead zone

$u_{\text{ref}} = 1V$

$u_{\text{min}} \sim 1/(2A)$

$A = 10^3 \sim 10^4$

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Decimation Filters for MOD1 (1)

- The sinc filter:

  Averaging over N samples (running-average)

  \[ w(n) = \frac{1}{N} \sum_{i=0}^{N-1} v(n - i) \]

  \[
  h_1(n) = \begin{cases} 
  1/N , & \text{if } (0 \leq n \leq N - 1) \\
  0 , & \text{otherwise}
  \end{cases}
  \]

  \[
  H_1(z) = \frac{1}{N} \frac{1 - z^{-N}}{1 - z^{-1}}
  \]

  \[
  H_1(e^{j2\pi f}) = \frac{\text{sinc}(Nf)}{\text{sinc}(f)}
  \]

  \[
  \text{sinc}(f) \triangleq \frac{\sin(\pi f)}{\pi f}
  \]
Decimation Filters for MOD1 (2)

- Responses:

\[ h_i(n) \]

\[ \frac{1}{N} \]

Normalized Frequency

\[ \text{OSR} = N \]

Areas around notches fold back to baseband after decimation if \( N = \text{OSR} \).

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Decimation Filters for MOD1 (3)

- Implementation:

\[ Q_1(z) = H_1(z) NTF(z) E(z) = \frac{1}{N} (1 - z^{-N}) E(z) \]

\[ q_1(n) = \frac{1}{N} [e(n) - e(n-N)] \]

Assuming \( e(n) \) and \( e(n-N) \) are uncorrelated:

**Inband noise after \( H_1 \):**

\[ \sigma^2_{q_1} = \frac{2\sigma^2_{r_{rms}}}{N^2} \]

**Inband noise before \( H_1 \):**

\[ \sigma^2_{q_0} = \frac{\pi^2 \sigma^2_e}{3N^3} \]

Total noise after \( H_1 \); Too much!  Total noise after ideal LPF; Much less than \( \sigma^2_{q_1} \)!
Decimation Filters for MOD1 (4)

- The sinc² filter:

\[ H_2(z) = \left( \frac{1 - z^{-N}}{N(1 - z^{-1})} \right)^2 \]

\[ H_2(e^{j2\pi f}) = \left( \frac{\text{sinc}(Nf)}{\text{sinc}(f)} \right)^2 \]

\[ Q_2(z) = NTF(z)H_2(z)E(z) = \frac{1}{N^2} \left( 1 - z^{-N} \right) \left( 1 - z^{-N} \right) E(z) = \frac{1}{N} H_1(z) \left[ (1 - z^{-N})E(z) \right] \]

\[ q_2(n) = \frac{1}{N^2} \sum_{i=0}^{N-1} \left[ e(n-i) - e(n-N-i) \right] \]

Total power \( \sigma^2_{q_2} = \frac{2N\sigma^2_e}{N^4} = \frac{2\sigma^2_e}{N^3} \)

\[ \sigma^2_{q_2} = \frac{\sigma^2}{3N^3} < \sigma^2_z \] (but signal is also reduced by \( H_2 \))

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April 2005
Decimation Filters for MOD1 (5)

- Response:

- Implementation:

\[(1 - z^{-1})^L\]

Or Hogencanuel

error!
References


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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>input step size (LSB size)</td>
<td>2</td>
</tr>
<tr>
<td>output step size</td>
<td>2</td>
</tr>
<tr>
<td>number of steps</td>
<td>$M = 2^{N-1}$</td>
</tr>
<tr>
<td>number of levels</td>
<td>$M + 1$</td>
</tr>
<tr>
<td>$\log_2$ = number of bits</td>
<td>$\lceil \log_2(M + 1) \rceil$</td>
</tr>
<tr>
<td>no-overload input range</td>
<td>$[-(M + 1), M + 1]$</td>
</tr>
<tr>
<td>full-scale</td>
<td>$2M$</td>
</tr>
<tr>
<td>input thresholds</td>
<td>$0, \pm 2, \ldots, \pm(M - 1), M$ odd</td>
</tr>
<tr>
<td></td>
<td>$\pm 1, \pm 3, \ldots, \pm(M - 1), M$ even</td>
</tr>
<tr>
<td>output levels</td>
<td>$\pm 1, \pm 3, \ldots, \pm M, M$ odd</td>
</tr>
<tr>
<td></td>
<td>$0, \pm 2, \pm 4, \ldots, \pm M, M$ even</td>
</tr>
</tbody>
</table>

Table 2.1. Properties of the symmetric quantizers of Figs. 2.3 and 2.4.