Nyquist-Rate D/A Converters

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D/A Converter Basics.

- $B_{in}$ is a digital signal (or word),
  
  $$B_{in} = b_1 2^{-1} + b_2 2^{-2} + \ldots + b_N 2^{-N}$$  
  \hspace{1cm} (1)

- $b_i$ equals a “1” or a “0” (i.e. a binary digit).

- $V_{ref}$ — an analog reference; $V_{out}$ — output.
  
  $$V_{out} = V_{ref}(b_1 2^{-1} + b_2 2^{-2} + \ldots + b_N 2^{-N})$$  
  \hspace{1cm} (2)

- **Define** $V_{LSB}$ to be LSB signal change, $V_{LSB} \equiv V_{ref}/2^N$
D/A Converter Basics

- For errors, define “units” of LSB $1 \text{LSB} = 1/2^N$

- A *multiplying* D/A allows $V_{\text{ref}}$ to be a *varying input* — $V_{\text{out}}$ proportional to multiplication of $V_{\text{ref}}$ and $B_{\text{in}}$.

- For *ideal* D/A, output signal is a *well defined value* — no quantization error!

\[
\frac{V_{\text{out}}}{V_{\text{ref}}} = \frac{1}{4} = 1 \text{ LSB}
\]

\[
\frac{V_{\text{LSB}}}{V_{\text{ref}}} = 1/4 = 1 \text{ LSB}
\]
D/A Resistor-String (Hamadé, JSSC, Dec. 1978)

- Guaranteed monotonic
- Integrated with better than 10-bits absolute accuracy.
- Delay through the switch network major speed limitation
- Resistors might be realized using polysilicon
- If n-channel only used, can be laid out small
- Requires $2^N$ resistors
D/A Resistor-String — Digital Decoding

- Higher speed implementation (less resistance thru transistors)
- Large cap load on buffer input
- Can pipeline digital decoding for faster speed
- Requires $2^N$ resistors

$V_{ref}$

$2^N$ Resistors

3 to 1 of 8 decoder

$b_1$

$b_2$

$b_3$

$V_{out}$

3-bit
Folded-resistor-string D/A

- (Abrial, JSSC, Dec. 1988)

- Less capacitance load over the single bus approach

- Requires $2^N$ resistors
Binary-Weighted Resistor D/A’s.

\[ V_{out} = -R_F V_{ref} \left( -\frac{b_1}{2R} - \frac{b_2}{4R} - \frac{b_3}{8R} - \cdots \right) \] (3)

- Only N resistors
- Resistor and current ratios are on the order of \(2^N\)
- No guarantee of monotonicity.
- Prone to glitches (more later).
Reduced Spread Binary Resistor D/A

- Reduced resistor spread
- Keep repeating this procedure —> R-2R ladder

\[ V_A = \frac{1}{4}(-V_{\text{ref}}) \]

4-bit
R-2R Based D/A Converters

\[ R_1 \quad R'_1 \quad R_2 \quad R'_2 \quad R_3 \quad R'_3 \quad R_4 \quad R'_4 \]

\[ V_{\text{ref}} \quad V_{\text{ref}} \quad V_{\text{ref}} \quad V_{\text{ref}} \]

\[ 2R \quad 2R \quad 2R \quad 2R \]

\[ \frac{V_{\text{ref}}}{2R} \quad \frac{V_{\text{ref}}}{4R} \quad \frac{V_{\text{ref}}}{8R} \quad \frac{V_{\text{ref}}}{16R} \]

\[ R'_4 = 2R \]
\[ R_4 = 2R \parallel 2R = R \]
\[ R'_3 = R + R_4 = 2R \]
\[ R_3 = 2R \parallel R'_3 = R \]

• Small size, good matching (only R and 2R)
R-2R Based Resistor Ladders

• Example D/A converter

![Diagram of R-2R ladder with switches and resistors labeled with currents and voltages.]

- Currents through the switches are scaled
- Should scale switch sizes for good accuracy
- No node voltage changes except for output —> fast

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R-2R Based Resistor Ladders

- Slower circuit having \textit{equal} current through switches

- Node voltages change — slower circuit
- No need to scale switch sizes (smaller size)
Glitches

- Different delays for switching the different currents
- MSB change often worst case

- Glitches can be minimized by limiting the bandwidth but that slows down circuit
- Use thermometer code to reduce glitches
Charge-Redistribution SC D/A’s

- Programmable SC gain amplifier.
- Sign bit realized by interchanging input phases.
- Carefully clock-waveforms required to minimize voltage dependency of clock-feed-through.
- Digital codes should be changed when input side of capacitors are connected to ground. Requires extra digital complexity.
# Thermometer D/A Converters

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Binary</th>
<th>Thermometer Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1</td>
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<tr>
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</tr>
<tr>
<td>7</td>
<td>1 1 1 1</td>
<td>1 1 1 1 1 1 1 1 1</td>
</tr>
</tbody>
</table>

Binary-to-thermometer code conversion
Thermometer Code D/A Converter

Top Capacitors are Connected to Ground

Bottom Capacitors are Connected to V_{ref}

- $2^N$ unit sized caps
- Guaranteed monotonic
- Much lower glitching
- Low DNL
Current-Mode D/A’s

- Thermometer-code
- High-speed, output feeds directly to resistor
- Important that delay to all the switches are equal.
- Overlapped clocks much better than having non-overlapped clocks.
Current-Steered D/A [Colles, 88]

- Operates as cascode current sources.
- For max speed, keep voltage swing at source of Q1 small (just turned off)
- Switching feed-through from the digital input enhances switching speed.
Segmented D/A

- Schoeff, 79; Saul, 85; Grebene, 84

- Combine thermometer and binary
- Accuracy needed for LSB reduced
- Glitches reduced
- Very popular
Dynamically-Matched Current Sources

• Schouwenaar, 88

- Each current source is calibrated with a single reference
- 64 used so D/A can continue operating
- Achieved 92 dB SNDR, and 20 mW with 3V.
- Used for audio application

• Dynamic technique with current switching for realizing very well-matched current sources

• Up to 16 bit accuracy
Dynamically-Matched Current Sources

- Minimize clock-feedthrough and charge-injection by having capacitance $C_{gs}$ and bias voltage $V_{GS}$ large.
- Implies voltage error causes less current deviation.

- Current source $0.9I$ added so a low $gm$ device used (W/L equal to 10/75).
- Re-calibrate before leakage causes 0.5LSB error.