CS331 (Spring 2017): Introduction to Artificial Intelligence
Written Assignment #2

Date handed out: April 28, 2017
Date due: May 5, 2017 at the start of class
Total: 31 points

The written portion of this assignment is to be done individually. Please hand in a hardcopy. Assignments done on a word processor are preferred but not mandatory. For hand written assignments, if we cannot read your writing, we cannot mark your assignment.

1. (From 7.4 in the book) For each of the following statements, prove if it is true or false.
   a) \((A \land B) \equiv (A \leftrightarrow B)\) [3 points]
   True because the left-hand side has exactly one model that is one of the two models of the right-hand side.

   b) \((C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))\) [3 points]
   True. You can prove this with truth table enumeration or logical equivalences:
   \((C \lor (\neg A \land \neg B)) \equiv (C \lor \neg A) \land (C \lor \neg B)\)  distributivity
   \equiv (A \Rightarrow C) \land (B \Rightarrow C)\)  implication elimination

   c) \((A \lor B) \land \neg(A \Rightarrow B)\) is satisfiable [3 points]
   \((A \lor B) \land \neg(A \Rightarrow B)\)  \equiv (A \lor B) \land \neg A \lor B
   \equiv (A \lor B) \land (A \lor \neg A)
   \equiv (A \land A \lor \neg B) \lor (B \land A \lor \neg B)
   \equiv (A \land \neg B) \lor false
   Satisfiable with (A \land \neg B)

2. (From 7.10 in the book) Decide whether each of the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11.
   a) \(Smoke \Rightarrow Smoke\) [2 points]
   \(\neg Smoke \lor Smoke = True\)
   Valid

   b) \((Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)\) [4 points]
   \(\neg (Smoke \lor Fire) \Rightarrow (\neg Smoke \lor \neg Fire)\)
   \(\equiv (\neg Smoke \lor Fire) \Rightarrow (Smoke \lor \neg Fire)\)
   \(\equiv (\neg Smoke \lor Fire) \lor (Smoke \lor \neg Fire)\)
   \(\equiv (Smoke \land \neg Fire) \lor (Smoke \lor \neg Fire)\)
   \(\equiv (Smoke \lor \neg Fire \lor Smoke) \land (\neg Fire \lor Smoke \lor \neg Fire)\)
\[
(Smoke \lor \neg Fire) \land (Smoke \lor \neg Fire)
\]
\[= Smoke \lor \neg Fire\]
Neither

c) \(Smoke \Rightarrow Fire \Rightarrow ((Smoke \land Heat) \Rightarrow Fire)\) [4 points]
\[= (\neg Smoke \lor Fire) \Rightarrow (\neg (Smoke \land Heat) \lor Fire)\]
\[= (\neg Smoke \lor Fire) \lor (\neg Smoke \lor \neg Heat \lor Fire)\]
\[= (Smoke \land \neg Fire) \lor (\neg Smoke \lor \neg Heat \lor Fire)\]
\[= (Smoke \land \neg Fire) \land (\neg Fire \lor (\neg Smoke \lor \neg Heat \lor Fire))\]
\[= true \land true = true\]
Valid

3. (Exercise 7.2 in the book which was adapted from Barwise and Etchemendy (1993)).

If a unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

a) Write down the knowledge base for the facts above in CNF (4 points)

Can you prove that the unicorn is:

b) Mythical? [4 points]
c) Magical? [4 points]
d) Horned? [4 points]

Mythical \(\Rightarrow\) \(\neg\)Mortal
\[\neg\text{Mythical} \Rightarrow \text{Mortal} \land \text{Mammal}\]
\[\neg\text{Mortal} \lor \text{Mammal} \Rightarrow \text{Horned}\]
Horned \(\Rightarrow\) Magical

\[\neg\text{Mythical} \lor \neg\text{Mortal}\]
\[\neg\neg\text{Mythical} \lor (\text{Mortal} \land \text{Mammal})\]
\[= \text{Mythical} \lor (\text{Mortal} \land \text{Mammal})\]
\[= (\text{Mythical} \lor \text{Mortal}) \land (\text{Mythical} \lor \text{Mammal})\]
\[\neg(\neg\text{Mortal} \lor \text{Mammal}) \lor \text{Horned}\]
\[= (\text{Mortal} \land \neg\text{Mammal}) \lor \text{Horned}\]
\[= (\text{Mortal} \lor \text{Horned}) \land (\neg\text{Mammal} \lor \text{Horned})\]
\[\neg\text{Horned} \lor \text{Magical}\]
a) CNF KB
\[\neg\text{Mythical} \lor \neg\text{Mortal}\]
\[\text{Mythical} \lor \text{Mortal}\]
\[\text{Mythical} \lor \text{Mammal}\]
Mortal \lor \text{Horned}
\neg \text{Mammal} \lor \text{Horned}
\neg \text{Horned} \lor \text{Magical}

b) \text{Does KB \models Mythical? No}
c) \text{Does KB \models Magical? Yes}
\neg \text{Magical}, \neg \text{Horned} \lor \text{Magical} = \neg \text{Horned}
\text{Mortal} \lor \text{Horned}, \neg \text{Horned} = \text{Mortal}
\neg \text{Mythical} \lor \neg \text{Mortal}, \text{Mortal} = \neg \text{Mythical}
\text{Mythical} \lor \text{Mammal}, \neg \text{Mythical} = \text{Mammal}
\neg \text{Mammal} \lor \text{Horned}, \text{Mammal} = \text{Horned}
\text{Horned} \lor \neg \text{Horned} = \{ \}
d) \text{Does KB \models Horned? Yes}
\text{Mortal} \lor \text{Horned}, \neg \text{Horned} = \text{Mortal}
\neg \text{Mythical} \lor \neg \text{Mortal}, \text{Mortal} = \neg \text{Mythical}
\text{Mythical} \lor \text{Mammal}, \neg \text{Mythical} = \text{Mammal}
\neg \text{Mammal} \lor \text{Horned}, \text{Mammal} = \text{Horned}