CS 331: Artificial Intelligence
Uninformed Search

Real World Search Problems
Simpler Search Problems

Assumptions About Our Environment

- Fully Observable
- Deterministic
- Sequential
- Static
- Discrete
- Single-agent
Search Problem Formulation

A search problem has 5 components:
1. A finite set of states \( S \)
2. A non-empty set of initial states \( I \subseteq S \)
3. A non-empty set of goal states \( G \subseteq S \)
4. A successor function \( \text{succ}(s) \) which takes a state \( s \) as input and returns as output the set of states you can reach from state \( s \) in one step.
5. A cost function \( \text{cost}(s,s') \) which returns the non-negative one-step cost of travelling from state \( s \) to \( s' \). The cost function is only defined if \( s' \) is a successor state of \( s \).

Example: Oregon

\[
S = \{ \text{Coos Bay, Newport, Corvallis, Junction City, Eugene, Medford, Albany, Lebanon, Salem, Portland, McMinnville} \}
\]
\[
I = \{ \text{Corvallis} \}
\]
\[
G=\{\text{Medford}\}
\]
\[
\text{Succ}(\text{Corvallis})=\{ \text{Albany, Newport, McMinnville, Junction City} \}
\]
\[
\text{Cost}(s,s') = 1 \text{ for all transitions}
\]
Results of a Search Problem

- Solution
  Path from initial state to goal state

- Solution quality
  Path cost (3 in this case)

- Optimal solution
  Lowest path cost among all solutions (In this case, we found the optimal solution)

Search Tree

Start with Initial State
Search Tree

Is initial state the goal?
- Yes, return solution
- No, apply Successor() function

Search Tree

Apply Successor() function

These nodes have not been expanded yet. Call them the fringe. We'll put them in a queue.

Queue

- McMinnville
- Albany
- Junction City
- Newport
Now remove a node from the queue. If it’s a goal state, return the solution. Otherwise, call Successor() on it, and put the results in the queue. Repeat.

Things to note:

- Order in which you expand nodes (in this example, we took the first node in the queue)
- Avoid repeated states
Tree-Search Pseudocode

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
    fringe ← INSERT(Make-Node(Initial-State[problem]), fringe)
loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem)(State[node]) then return SOLUTION(node)
    fringe ← INSERT-ALL(Expand(node, problem), fringe)

function Expand(node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-Fn(problem)(State[node]) do
        s ← a new NODE
        Parent-Node[s] ← node; Action[s] ← action; State[s] ← result
        Path-Cost[s] ← Path-Cost[node] + Step-Cost(node, action, s)
        Depth[s] ← Depth[node] + 1
        add s to successors
    return successors
```

Note: Goal test happens after we grab a node off the queue.
Tree-Search Pseudocode

Why are these parent node backpointers important?

function `Tree-Search(problem, fringe)` returns a solution, or failure
fringe ← `Insert(Make-Node(INITIAL-STATE[problem]), fringe)`
loop do
  if fringe is empty, then return failure
  node ← `First(fringe)`
  fringe ← `Delete-First(fringe)`
  if `Is-Solution(node)` then return `Solution(node)`
  successors ← `Expand(node, problem)`
  fringe ← `Insert(Make-Node(node), fringe)`
  for each `s` in successors do
    if `s` is not in fringe then
      fringe ← `Insert(Make-Node(s), fringe)`
  end loop

function `Expand(node, problem)` returns a set of nodes
successors ← the empty set
for each `action, result` in `SUCCESSOR-Fn(problem)(STATE[node])` do
  `s` ← a new NODE
  `PARENT-Node[s]` ← `node`, `ACTION[s]` ← `action`, `STATE[s]` ← `result`
  `PATH-COST[s]` ← `PATH-COST[node] + STEP-COST(node, action, s)`
  `DEPTH[s]` ← `DEPTH[node] + 1`
  add `s` to `successors`
return `successors`

Uninformed Search

- No info about states other than generating successors and recognizing goal states
- Later on we’ll talk about informed search – can tell if a non-goal state is more promising than another
Evaluating Uninformed Search

- Completeness
  Is the algorithm guaranteed to find a solution when there is one?
- Optimality
  Does it find the optimal solution?
- Time complexity
  How long does it take to find a solution?
- Space complexity
  How much memory is needed to perform the search

Complexity

1. Branching factor (b) – maximum number of successors of any node
2. Depth (d) of the shallowest goal node
3. Maximum length (m) of any path in the search space

Time Complexity: number of nodes generated during search
Space Complexity: maximum number of nodes stored in memory
Uninformed Search Algorithms

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative Deepening Depth-first Search
- Bidirectional search

Breadth-First Search

- Expand all nodes at a given depth before any nodes at the next level are expanded
- Implement with a FIFO queue
Breadth First Search Example

Not yet reached
Expanded nodes on current path
On fringe but unexpanded
Current node to be expanded

Breadth First Search Example

Not yet reached
Expanded nodes on current path
On fringe but unexpanded
Current node to be expanded
## Evaluating BFS

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Complete?</strong></td>
<td>Yes provided branching factor is finite</td>
</tr>
<tr>
<td><strong>Optimal?</strong></td>
<td>Yes if step costs are identical</td>
</tr>
<tr>
<td><strong>Time Complexity</strong></td>
<td>$b+b^2+b^3+\ldots+b^d+(b^{d+1}-b)=O(b^{d+1})$</td>
</tr>
<tr>
<td><strong>Space Complexity</strong></td>
<td>$O(b^{d+1})$</td>
</tr>
</tbody>
</table>

Exponential time and space complexity make BFS impractical for all but the smallest problems.
Uniform-cost Search

- What if step costs are not equal?
- Recall that BFS expands the shallowest node
- Now we expand the node with the lowest path cost
- Uses priority queues

Note: Gets stuck if there is a zero-cost action leading back to the same state.
For completeness and optimality, we require the cost of every step to be $\geq \varepsilon$.

Evaluating Uniform-cost Search

<table>
<thead>
<tr>
<th>Complete?</th>
<th>Yes provided branching factor is finite and step costs $\geq \varepsilon$ for small positive $\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal?</td>
<td>Yes</td>
</tr>
<tr>
<td>Time Complexity</td>
<td>$O(b^{1+\text{floor}(C*/\varepsilon)})$ where $C^*$ is the cost of the optimal solution</td>
</tr>
<tr>
<td>Space Complexity</td>
<td>$O(b^{1+\text{floor}(C*/\varepsilon)})$ where $C^*$ is the cost of the optimal solution</td>
</tr>
</tbody>
</table>
Depth-first Search

- Expands the deepest node in the current fringe of the search tree
- Implemented with a LIFO queue

Depth-first Search Example
Depth-first Search Example

Evaluating Depth-first Search

<table>
<thead>
<tr>
<th>Complete?</th>
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<tbody>
<tr>
<td>Optimal?</td>
<td></td>
</tr>
<tr>
<td>Time Complexity</td>
<td></td>
</tr>
<tr>
<td>Space Complexity</td>
<td></td>
</tr>
</tbody>
</table>
Evaluating Depth-first Search

<table>
<thead>
<tr>
<th>Complete?</th>
<th>No (Might not terminate if it goes down an infinite path with no solutions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal?</td>
<td>No (Could expand a much longer path than the optimal one first)</td>
</tr>
<tr>
<td>Time Complexity</td>
<td>$O(b^m)$</td>
</tr>
<tr>
<td>Space Complexity</td>
<td>$O(bm)$</td>
</tr>
</tbody>
</table>

Depth-limited Search

- Solves infinite path problem by using predetermined depth limit $l$
- Nodes at depth $l$ are treated as if they have no successors
- Can use knowledge of the problem to determine $l$ (but in general you don’t know this in advance)
Evaluating Depth-limited Search

<p>| | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Complete?</td>
<td>No (If shallowest goal node beyond depth limit)</td>
</tr>
<tr>
<td>Optimal?</td>
<td>No (If depth limit &gt; depth of shallowest goal node and we expand a much longer path than the optimal one first)</td>
</tr>
<tr>
<td>Time Complexity</td>
<td>$O(b^l)$</td>
</tr>
<tr>
<td>Space Complexity</td>
<td>$O(b^l)$</td>
</tr>
</tbody>
</table>

Iterative Deepening Depth-first Search

- Do DFS with depth limit 0, 1, 2, … until a goal is found
- Combines benefits of both DFS and BFS
Iterative Deepening Depth-first Search Example

Limit = 0

Limit = 1

Limit = 2

IDDFS Example

Limit = 3
IDDFS Example

Limit = 3 (Continued)

Evaluating Iterative Deepening
Depth-first Search

<table>
<thead>
<tr>
<th>Complete?</th>
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<tbody>
<tr>
<td>Optimal?</td>
<td></td>
</tr>
<tr>
<td>Time Complexity</td>
<td></td>
</tr>
<tr>
<td>Space Complexity</td>
<td></td>
</tr>
</tbody>
</table>
Evaluating Iterative Deepening Depth-first Search

<table>
<thead>
<tr>
<th>Complete?</th>
<th>Yes provided branching factor is finite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal?</td>
<td>Yes if the path cost is a nondecreasing function of the depth of the node</td>
</tr>
<tr>
<td>Time Complexity</td>
<td>O(b^d)</td>
</tr>
<tr>
<td>Space Complexity</td>
<td>O(bd)</td>
</tr>
</tbody>
</table>

Isn’t Iterative Deepening Wasteful?

- Actually, no! Most of the nodes are at the bottom level, doesn’t matter that upper levels are generated multiple times.
- To see this, add up the 4th column below:

<table>
<thead>
<tr>
<th>Depth</th>
<th># of nodes</th>
<th># of times generated</th>
<th>Total # of nodes generated at depth d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>b</td>
<td>d</td>
<td>(d)b</td>
</tr>
<tr>
<td>2</td>
<td>b^2</td>
<td>d-1</td>
<td>(d-1)b^2</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>d</td>
<td>b^d</td>
<td>1</td>
<td>(1)b^d</td>
</tr>
</tbody>
</table>
Is Iterative Deepening Wasteful?

Total # of nodes generated by iterative deepening:

\[(d)b + (d-1)b^2 + \ldots + (1)b^d = O(b^d)\]

Total # of nodes generated by BFS:

\[b + b^2 + \ldots + b^d + (b^{d+1} - b) = O(b^{d+1})\]

In general, iterative deepening is the preferred uninformed search method when there is a large search space and the depth of the solution is not known.

Bidirectional Search

- Run one search forward from the initial state
- Run another search backward from the goal
- Stop when the two searches meet in the middle
Bidirectional Search

• Needs an efficiently computable Predecessor() function
• What if there are several goal states?
  – Create a new dummy goal state whose predecessors are the actual goal states
• Difficult when the goal is an abstract description like “no queen attacks another queen”

Evaluating Bidirectional Search

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Complete?</td>
<td>Yes provided branching factor is finite and both directions use BFS</td>
</tr>
<tr>
<td>Optimal?</td>
<td>Yes if the step costs are all identical and both directions use BFS</td>
</tr>
<tr>
<td>Time Complexity</td>
<td>(O(b^{d/2}))</td>
</tr>
<tr>
<td>Space Complexity</td>
<td>(O(b^{d/2})) (At least one search tree must be kept in memory for the membership check)</td>
</tr>
</tbody>
</table>
Avoiding Repeated States

- Tradeoff between space and time!
- Need a closed list which stores every expanded node (memory requirements could make search infeasible)
- If the current node matches a node on the closed list, discard it (i.e. discard the newly discovered path)
- We’ll refer to this algorithm as GRAPH-SEARCH
- Is this optimal? Only for uniform-cost search or breadth-first search with constant step costs.

GRAPH-SEARCH

```
function GRAPH-SEARCH(problem, fringe) returns a solution, or failure
  closed ← an empty set
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem)(STATE[node]) then return SOLUTION(node)
    if STATE[node] is not in closed then
      add STATE[node] to closed
      fringe ← INSERT-ALL(EXPAND(node, problem), fringe)
```

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Things You Should Know

• How to formalize a search problem
• How BFS, UCS, DFS, DLS, IDS and Bidirectional search work
• Whether the above searches are complete and optimal plus their time and space complexity
• The pros and cons of the above searches