1 Problem 1

A parking lot has 5 parking spaces and there is a camera to monitor parking cars. The camera is set up at height of 10 meters and each parking space is 3 meters long (see in the figure below). Suppose the parking lot is full and one car can only park in one parking spot. To save energy, the camera operates every half an hour (e.g., 10:00pm, 10:30pm, 11:00pm, …) and a camera’s angle $\theta$ is picked at random according to the uniform distribution ($\theta \sim U[0, \pi]$).

(a) Calculate the probability that a car at parking spot $k$ ($k = 1, 2, \ldots, 5$) will be detected by camera in one snapshot.

(b) What parking spot you think has highest detection probability? Explain.

(c) Suppose the car in the 2nd slot is stolen at 9:59pm, what is the probability that the missing car will be detected by the camera by 6:00am tomorrow (not including 6:00am)? (Hints: Use the Geometric distribution)

(d) To guarantee a probability of detecting the missing car by 6:00am (not including 6:00am) at least 95%, how often should the camera be used instead of half an hour? Let the first time the camera activates after missed the car missed be 10:00pm.

2 Problem 2

Let the number of bits received at the receiver follow a Poisson distribution with $\lambda = 10$ bits per $1\mu s$.

(a) What is the probability that the receiver receives more than 13 bits in $1\mu s$?

(b) What is the probability that the receiver receives more than 130 bits in $10\mu s$?

(c) Use MATLAB to compute the results of (a) and (b). Provide your MATLAB code in the submission. (Hints: You can use MATLAB functions: $\text{gammaln.m}$ and/or $\text{factorial.m}$ to compute the factorials.)
3 Problem 3

A mouse starts from the root of the tree and climbs up the tree. At each node, there are three branches, the mouse walks to the center branch with probability $2/3$ and to the right with probability $1/12$, and to the left with probability $1/4$. The mouse stops whenever it is trapped on a left or right branch. What is the probability of the mouse being trapped at level $k$ ($k = 1, 2, \ldots$) of the tree.

4 Problem 4

A set of 1000Ω resistors are produced with 5% tolerance (i.e., the resistor is rejected if its resistance is not within ±5%). Let $X$ be the resistance of a resistor. Assume that $X$ is a normal random variable with mean 1000 and variance 625. What is the probability that a resistor picked at random will be rejected?