Question 1 [5 marks]:
Consider a resistor, with cross-sectional area $A$ and length $L$, as shown in the figure below. Assume the resistor is made of a material of resistivity $\rho$, and current flows in the direction indicated by the red arrow.

Using the following five equations, derive an expression for the resistance of resistor ($R$) in terms of its resistivity ($\rho$), cross-sectional area ($A$) and length ($L$) only.

\[ E = \frac{V}{L} \quad (1) \]

- $E$: Electric field strength.
- $V$: Voltage applied.
- $L$: Distance voltage is applied over.

\[ J = \frac{I}{A} \quad (2) \]

- $J$: Current density.
- $I$: Current.
- $A$: Cross-sectional area through-which current flows.

\[ V = IR \quad (3) \]

- $V$: Applied voltage
- $I$: Current.
- $R$: Resistance.

\[ J = \sigma E \quad (4) \]

- $J$: Current density.
- $\sigma$: Conductivity.
- $E$: Electric Field Strength.
\[ \rho = \frac{1}{\sigma} \]  

- \( \rho \): Resistivity.
- \( \sigma \): Conductivity.

**Question 2 [5 marks]:**
The charge carrier mobility in silicon depends on doping concentration. There is an empirical relationship that has been reported \([1]\) to relate doping concentration and carrier mobility:

\[ \mu = \mu_{\text{min}} + \frac{\mu_{\text{max}} - \mu_{\text{min}}}{1 + \left( \frac{n}{n_r} \right)^\alpha} \]

Where: \( \mu \) is the carrier mobility in the wafer, \( n \) is the dopant concentration, and the remaining parameters: \( \mu_{\text{max}}, \mu_{\text{min}}, n_r \) and \( \alpha \), are all fitting parameters of the empirical relationship. Phosphorus is a common \( n \)-type dopant (donates electrons) in silicon, and boron is a common \( p \)-type dopant (donates holes). The parameters for phosphorus and boron doping of silicon, at room-temperature, are given in the table below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Phosphorus</th>
<th>Boron</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{\text{min}} )</td>
<td>68.5 cm(^2)/Vs</td>
<td>44.9 cm(^2)/Vs</td>
</tr>
<tr>
<td>( \mu_{\text{max}} )</td>
<td>1414 cm(^2)/Vs</td>
<td>470.5 cm(^2)/Vs</td>
</tr>
<tr>
<td>( n_r )</td>
<td>( 9.20 \times 10^{16} ) cm(^{-3})</td>
<td>( 2.23 \times 10^{17} ) cm(^{-3})</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.711</td>
<td>0.719</td>
</tr>
</tbody>
</table>

If we make the approximation that every dopant atoms provides one electron (for phosphorus) or one hole (boron), calculate the room-temperature resistivity of a silicon wafer uniformly doped with \( 1 \times 10^{17} \) cm\(^{-3}\) of phosphorus atoms and \( 1 \times 10^{16} \) cm\(^{-3}\) of boron atoms.

**Question 3 [9 marks]:**
The figure below shows a schematic of a typical 4-probe measurement. Current is driven between the two outer probes (1 & 4) and the voltage drop is measured across the central two probes (2 & 3). The probes are separated by distances \( S_1, S_2, S_3 \).
a) We apply a current of 100μA across a 200nm thick sample of indium tin oxide (ITO), with a resistivity of $\rho = 7.2 \times 10^{-4} \Omega \text{cm}$. What voltage drop would we expect to measure? Assume the probes are equally spaced: $S = S_1 = S_2 = S_3$, and that the sample thickness $t \ll S/2$. [3 marks]

b) Next, we wish to measure a (different) sample that we approximate as infinitely thick ($t \gg S$). If the probes are equally spaced ($S = S_1 = S_2 = S_3$), what separation is required to evaluate the resistivity $\rho$ in $\Omega \text{cm}$ by simply taking the ratio of the measured voltage (in volts) to applied current (in amps)? [3 marks]

c) Now consider the case where we again measure a sample that we can approximate as infinitely thick, but now the probes are not equally spaced (i.e. $S_1 \neq S_2 \neq S_3$). The probe separations are as follows: $S_1 = 10\mu\text{m}$, $S_2 = 50\mu\text{m}$, $S_3 = 200\mu\text{m}$. If a current of 100 nA is driven across the outer probes (1 & 4), and we measure a voltage of 859 mV across the inner two probes (2 & 3) in this setup, what is the resistivity of this material? Give your answer in either $\Omega \text{m}$ or $\Omega \text{cm}$. [3 marks]

Question 4 [6 marks]:
A common way to evaluate contact resistance is to use a transfer-length measurement, as shown in the figure below.

![Diagram](image)

The resistance as a function of channel length for such an experiment is given in the table below. Using this data, approximate the contact resistance for this system. Values $\pm 50\%$ of the real answer will be accepted.

<table>
<thead>
<tr>
<th>Distance (μm)</th>
<th>Measured Total Resistance (MΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.0</td>
</tr>
<tr>
<td>20</td>
<td>3.0</td>
</tr>
<tr>
<td>40</td>
<td>5.0</td>
</tr>
<tr>
<td>80</td>
<td>7.0</td>
</tr>
<tr>
<td>160</td>
<td>9.0</td>
</tr>
</tbody>
</table>