Lecture 10
Charge Carrier Mobility
Schroder: Chapter 8

Announcements

Homework 2/6:
• Is online now.
• Due Today.
• I will return it next monday (7th May).

Midterm Exam:
• Friday May 4th at 10:00am in STAG113
• Exam will last 45 minutes.
  • The exam will start exactly at 10:00am!
• Closed book and closed notes.
• Review lecture on Wednesday will cover examples and more details.
Lecture 10

- Overview of Mobility.
- Hall Measurements.
- van der Pauw Method.
- AC Hall Measurements.
- Time-of-Flight Mobility.

Overview of Mobility
Mobility

• We talked a bit about mobility in Lecture 2.
• It is basically just the velocity at which a carrier moves in a semiconductor, normalized for electric field:
  \[ \mu = \frac{\nu}{E} \]

• Since carriers reach terminal velocity very quickly (in most systems) we are do not consider acceleration in our formulism.

Mobility

• Normally the mobilities for electrons and holes are dissimilar: \( \mu_e \neq \mu_h \).
• Some compounds are very anisotropic:
• Temperature is also a huge factor:

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Why is it Important?

- Transistors:
  - The speed at which transistors can switch states → circuits process information is highly dependent upon mobility.
  - The mobility also quantifies the amplification properties of a transistor.
  - The gradual channel approximation:

\[
I_D = \frac{W}{L} \mu C_{ox} \left( (V_G - V_T) V_D - \frac{V_D^2}{2} \right)
\]

- Solar Cells
  - When carriers are created in a solar cell, we need them to exit the device before they recombine.
  - The distance they can move (on average) is described by the diffusion length:

\[
L_e = \sqrt{\frac{k_B T}{e} \tau_e \mu_e} \quad L_h = \sqrt{\frac{k_B T}{e} \tau_h \mu_h}
\]

- \( \mu \) is carrier mobility.
- \( \tau \) is carrier lifetime.
What Determines Mobility

- Recall from Lecture 1 that Ohm’s Law is given by:

  \[ J = \sigma E \]

- Where the conductivity (\(\sigma\)) can be expressed as:

  \[ \sigma = q(\mu_n n + \mu_p p) \]

- Also recall that the essential physics determining mobility is given by:

  \[ \mu = \frac{q \langle \tau \rangle}{m^*} \]

- \(\langle \tau \rangle\) is the mean scattering time.
  - Determined primarily by the temperature and the ionized impurity concentration

Effective Mass

- Effective mass (\(m^*\)) comes up often in the discussion of mobility.
  - Basically it is a formalism which allows us use Newtonian laws of motion when describing a particle (electron or hole) in a solid.
  - We just describe the mass of the electron (or hole) as having a different mass to what it would have in a vacuum.

  \[ m \rightarrow m^* \]
Effective Mass

- A note about this:
- This approximation is derived\cite{1} using the following relationship between band energy and momentum:

\[ m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \]

\[ m^*_{ij} = \frac{\hbar^2}{\left(\frac{\partial^2E}{\partial k_i \partial k_j}\right)} \]

- This assumes that bands are parabolic.
- This is, at best, normally only locally true near maxima / minima.

\[ E \text{ [eV]} \]

\[ \begin{array}{c}
\text{Band Gap}
\end{array} \]

[1] Solid State Physics, Hook and Hall, Wiley

Measuring Mobility

- Transport measurements typically involve assessment of quantities such as current, resistance, conductance, etc., all of which involve a product of carrier density and velocity (i.e. mobility).
- For example, suppose that you measured the resistance of an n-type semiconductor and calculated its mobility.

\[ R = \frac{V}{I} = \frac{\rho L}{A} = \frac{L}{\sigma A} \]

\[ \sigma = q(\mu_n n + \mu_p p) \]

\[ \mu_n = \frac{\sigma}{q n} \]

\[ \sigma = q\mu_n n \]
Measuring Mobility

- This last result tells us that if we know the carrier concentration, $n$, we can find the mobility for this semiconductor.

$$\mu_n = \frac{\sigma}{qn}$$

- In general, $n$ is not known.
- Thus, in order to learn something about the mobility, we need to be able to separate carrier density from the relevant transport parameter, i.e., mobility.

Hall Measurements
Hall Measurements

- Hall Measurements allow independent determination of carrier density and mobility.
- It is hence a very valuable tool in characterization.
- Normally only useful for samples with mobilities > 1 cm²/Vs.
  - For lower mobility we need to use AC Hall.
  - Also, only works for unipolar samples. Either:
    - $n \gg p$, or:
    - $p \gg n$.

Consider a cuboid block of your material.

- We say it has dimensions of $L = \text{length}$, $W = \text{width}$, and $d = \text{depth}$.
- We will consider a p-type material, with no mobile electrons.
Hall Measurements

- We apply a voltage across the length of our block.
- We label it $V_\rho$ as in Schroder. Sometimes called $V_x$.
- We then measure the current that flows under this voltage in the x-direction.

\[ \rho = \frac{WdV_\rho}{Ll} \]

Current flows from positive to negative. So do holes.
Hall Measurements

- Next we apply a magnetic field in the \( z \)-direction.
- To understand how these carriers respond, we consider the Lorentz Force:

\[
\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})
\]

The Lorentz Force

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\[
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\]

- \( \mathbf{F} \) is the force the charge particle experiences.
- \( q \) is the charge on the particle (signed).
- \( \mathbf{E} \) is the electric field vector.
- \( \mathbf{v} \) is the velocity of our carrier.
- \( \mathbf{B} \) is the magnetic field vector.
The Lorentz Force

• All vectors are defined to be along axes in this arrangement:

\[
\begin{align*}
\vec{E} &= (E_x, 0, 0) \\
\vec{B} &= (0, 0, B_z) \\
\vec{v} &= (v_x, 0, 0)
\end{align*}
\]

The magnetic field is defined as being in the \( z \) direction only.

Initially the carriers are just traveling along the \( x \)-axis.

Electric field is applied voltage.

The force experienced by the carriers is:

\[
\vec{F} = q(\vec{E} + \vec{v} \wedge \vec{B})
\]

The cross-product of two vectors, \( \vec{a}, \vec{b} \) is:

\[
\vec{a} \wedge \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}
\]

Where:

• \( \theta \) is the angle between the vectors.

• \( \hat{n} \) is the unit vector perpendicular to \( \vec{a} \) and \( \vec{b} \).


"Right-hand rule"
**Cross Product Direction**

- Wikipedia has some good images which describe the direction of the unit vector.
- Note Wikipedia uses \( \times \) rather than \( \wedge \) to denote cross product.

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**The Lorentz Force**

\[
v \wedge B = |v||B| \sin \theta \hat{n}
\]

- Magnitudes are trivial for uniaxial vectors:
  \[
v = (v_x, 0, 0) \quad B = (0, 0, B_z)
  \]
  \[
  |v| = v_x \quad |B| = B_z
  \]
  \[
v \wedge B = |v||B| \sin \theta \hat{n} \quad v \wedge B = v_x B_z \sin \theta \hat{n}
  \]
- Since initially the carriers in our semiconductor are traveling perpendicular to the electric field, \( \theta = 90^\circ \).
  \[
  \sin(90^\circ) = 1 \quad v \wedge B = v_x B_z \hat{n}
  \]
The Lorentz Force
\[ \mathbf{v} \times \mathbf{B} = v_x B_z \hat{n} \]

- Finally, we just determine the direction of the unit vector (\( \hat{n} \)) using the right-hand-rule

\[ v = (v_x, 0, 0) \]
\[ \mathbf{B} = (0, 0, B_z) \]

\[ \mathbf{v} \times \mathbf{B} = v_x B_z \hat{y} = (0, v_x B_z, 0) \]

The Lorentz Force
\[ \mathbf{v} \times \mathbf{B} = v_x B_z \hat{y} = (0, v_x B_z, 0) \]

- Back to the Lorentz Force:
\[ \mathbf{F} = q(E + \mathbf{v} \times \mathbf{B}) \]
- Recall:
\[ \mathbf{E} = (E_x, 0, 0) \]
- So the force particles experience would be:
\[ \mathbf{F} = (qE_x, qv_x B_z, 0) \]
- We are interested in the deflection of the particles in the \( y \)-direction:
\[ F_y = qv_x B_z \]
Hall Voltage

- So when the magnetic field is applied the charges are accelerated in the y-direction.
- Carriers therefore build up on this side of the sample, giving rise to a potential gradient (i.e. a voltage).
- We can measure this Hall voltage, $V_H$.

$\begin{align*}
\text{Hall Voltage} \\
\text{Wikipedia has a very nice animation:} \\
\text{Janet Tate’s Lab (Physics) has a nice document on Hall Measurements:} \\
\end{align*}$

https://en.wikipedia.org/wiki/Hall_effect

The Hall Constant

• We define the Hall Constant:

\[ R_H = \frac{V_H d}{IB_Z} \]

• Once you have measured the resistivity and Hall constant (via Hall voltage), you can assess the carrier concentration and mobility.

• For notational ease we will henceforth say:

\[ |B| = B_Z = B \]

The Hall Constant

• It turns out\(^1\) that the Hall Constant is also equal to:

\[ R_H = \frac{(p - b^2 n) + (\mu_n B)^2 (p - n)}{q[(p + bn)^2 + (\mu_n B)^2 (p - n)^2]} \]

• Where:
  • \( p \) is delocalized hole density.
  • \( n \) is delocalized electron density.
  • \( \mu_n \) is electron mobility (also often labeled \( \mu_e \)).
  • \( \mu_p \) is hole mobility (also often labeled \( \mu_h \)).

\[ b = \frac{\mu_n}{\mu_p} \]

\(^1\) R.A. Smith, Semiconductors, Cambridge University Press, (1959)
The Hall Constant

\[ R_H = \frac{(p - b^2n) + (\mu_n B)^2(p - n)}{q[(p + bn)^2 + (\mu_n B)^2(p - n)^2]} \]

- This is the general form of \( R_H \), true for ambipolar / bipolar semiconductors.
- For unipolar semiconductors we can say:

- P-type: \( R_H (p \gg n) = + \frac{1}{qp} \)

- N-type: \( R_H (n \gg p) = - \frac{1}{qn} \)

Thus, the carrier concentration is obtained directly from the magnitude of the Hall constant.
- Additionally, the sign of the Hall constant establishes the carrier type, "+" for \( p \)-type and "−" for \( n \)-type.
- It is worth being aware that the Lorentz force leads to electron pile-up in an \( n \)-type semiconductor at the same interface as holes in a \( p \)-type semiconductor.
- In the Lorentz equation both \( v_x \) and \( q \) have their sign switched between holes and electrons.
- This leads to the Hall voltage polarity being switched.
The Hall Mobility

• With the Hall constant evaluated, we now know the carrier type and concentration.

\[ R_H(p \gg n) = + \frac{1}{qp} \quad R_H(n \gg p) = - \frac{1}{qn} \]

• Combined with our initial measurement of resistivity (in the absence of the \(B\)-field) we can evaluate the Hall mobility:

\[ \mu_H = \frac{|R_H|}{\rho} \]

• We call it \(\mu_H\) to denote that is was evaluated via a Hall measurement (not field-effect mobility).

The Hall Factor

• Before we finish this section, we must state that the Hall mobility we derived so far is for a very pure semiconductor, or one being measured under very high fields.

• More generally, we have to introduce a correction factor to our derived expressions.

• This is the Hall factor:

\[ r = \frac{\langle \tau^2 \rangle}{\langle \tau \rangle^2} \]

• \(\tau\) is the time between scattering events.
The Hall Factor

- The Hall factor accounts for the fact that electron and hole distributions are not mono-energetic, as assumed in derivations of previous equations.
- It turns out that: \(1 \leq r \leq 2\).
  - \(r = 1\) for very pure samples.
  - \(r = 1.18\) for phonon scattering.
  - \(r = 1.93\) for ionized impurity scattering.

\[ \mu_p = \frac{\mu_H}{r} \]
\[ \mu_n = \frac{\mu_H}{r} \]

\[ p = + \frac{r}{qR_H} \]
\[ n = - \frac{r}{qR_H} \]
Example Systems

• Dynacool can go down to $T \sim 2K$.

van der Pauw Method
van der Pauw Method

• The Hall technique basically requires a 3-dimensional sample:

• Sometimes we measure very thin films (~100’s nm).
• It would be impossible to contact such a sample from the side.

van der Pauw Method

• In Lecture 3 we talked about measuring the resistivity of arbitrarily shaped samples.
• For example:

• It turns out we can evaluate the mobility and carrier density in such a sample by applying a magnetic field across it.
Assumptions

• The formalism was derived by van der Pauw, but we will just take the result.

• For this analysis to be correct, we require:
  1. The contacts are at the edge of the sample.
  2. The contacts are sufficiently small.
  3. The sample is uniformly thick.
  4. The surface of the sample does not have any isolated holes.

van der Pauw Resistivity

• The resistivity is given by:
  \[ \rho = \frac{\pi}{\ln 2} d \frac{R_{12,34} + R_{23,41}}{2} F_v \]

• \( F_v \): A correction factor associated with arbitrary geometry.

• The \( R \) parameters are resistances measured across different pairs of terminals.
  \[ R_{12,34} = \frac{V_{34}}{I_{12}} \]
  \[ R_{23,41} = \frac{V_{41}}{I_{23}} \]

• \( d \): Sample thickness (note we used \( t \) in Lecture 3.)
Correction Factor

- The correction factor is evaluated by using the following relationship:

\[
\frac{R_r - 1}{R_r + 1} = \frac{F_v}{\ln 2} \arccosh \left( \frac{\exp[\ln(2)/F_v]}{2} \right)
\]

- Hence if \( R_{12,34} = R_{23,41} \), \( F_v = 1 \).
- For homogenous and isotropic samples this can be achieved by ensuring the path lengths 1→2 and 3→4 are equal.

Typical Geometries

- From Schroder Figs. 8.3 (p. 470), 8.8 (p. 476), or 8.9 (p. 476):
van der Pauw Hall Factor

• Due to the magnetoresistive effect, the resistivity of the sample will change under the influence of a magnetic field ($B$):

\[ \Delta R_{24,13} = R_{24,13}(B) - R_{24,13}(0) \]

• For the van der Pauw method the Hall factor is then given by:

\[ R_H = \frac{d\Delta R_{23,41}}{B} \]

• Ideally $R_H \neq R_H(B)$.

AC Hall Measurements
Low Mobility Samples

- We mentioned at the start of the lecture that DC Hall (i.e. the technique we have been talking about so far) is effective in measuring samples with carrier mobilities $\geq 1 \text{ cm}^2/\text{Vs}$.
- If we want to measure samples with lower mobility, or to measure mobility with better resolution, we need to use AC Hall measurements.
- AC measurements allows us to use lock-in techniques to greatly improve resolution down to $\sim 10^{-3} \text{ cm}^2/\text{Vs}$.

Sources of Error

- In an ideal situation the Hall voltage is as follows:
  \[
  V_H = \frac{R_H IB}{d}
  \]

- However, when we carry out a measurement in reality, what we measure is:
  \[
  V_m = V_H + V_0 + V_{TE}
  \]
Sources of Error

• The misalignment voltage $V_0$ is a small linear value that depends on the resistivity of the sample ($\rho$) and the measurement system.

• The thermoelectric voltage $V_{TE}$ arises from contacts between two different materials. It is independent of the current, but it does depend on any thermal gradients present.

• Neither of these voltages depend on the magnetic field strength: $V_0 \neq V_0(B)$ and $V_{TE} \neq V_{TE}(B)$.
  • This is important.

Sources of Error

• For high mobility samples:
  $$V_H \gg V_0 \quad V_H \gg V_{TE}$$

• And we can hence make the approximation that what we measure is the Hall Voltage:
  $$V_m \approx V_H$$

• However, at lower mobilities ($< 1 \text{ cm}^2/\text{Vs}$) we can no longer hold this assumption as generally true, and the analysis is no longer valid.
AC Magnetic Field

- We mentioned that neither the misalignment voltage or the thermoelectric voltage depend on the magnetic field strength, while the Hall voltage does:

\[ V_0 \neq V_0(B) \quad V_{TE} \neq V_{TE}(B) \quad V_H(B) = \frac{RHB}{d} \]

- We can design the experiment such that apply a sinusoidal magnetic field (AC) rather than a static field:

\[ B(t) = B_0 \sin(\omega t) \]

- In this case the measured voltage should be:

\[ V_m = V_H + V_0 + V_{TE} = \frac{RHB}{d} + V_0 + V_{TE} \]

\[ V_m(t) = \frac{RHB_0}{d} \sin(\omega t) + V_0 + V_{TE} \]

- I.e. if we only measure the time-dependent signal we can remove \( V_0 \) and \( V_{TE} \).

- This can be achieved with a lock-in amplifier.
Lock-In Techniques

- Lock in amplification techniques are an incredibly powerful way to improve signal to noise in any experiment that can be oscillated.
- Basically it is an amplifier that will accept signals of a certain frequency, and accept all others.

In reality you define a target frequency $\omega_0$, some bandwidth $\Delta \omega$ and then apply some attenuation factor that more heavily attenuates the signal the further from $\omega_0$ you are.

- Normally you chose a frequency ($f = \omega / 2\pi$) that is a prime number to avoid multiples of common noise signals (60Hz etc).
- Note for very high field magnets ($B \gg 1T$) you cannot switch the direction easily. For these measurements you can rotate the sample to get the same effect.
Time-of-Flight Drift Mobility

• The final technique we are going to talk about today is time of flight mobility.
• These techniques are still popular today, even though the original Haynes-Shockley experiment upon which it is based goes back to 1951.
• The idea is very simple:
  • Establish an electric field in the semiconductor
  • Inject carriers, let them drift in the electric field, and detect them down field.
  • If the drift distance is \( d \) and the drift time is \( t_d \), the drift velocity is given by \( v = d/t_d \).
Time-of-Flight Drift Mobility

- If the electric field \( (E) \) is small enough that linear transport holds, we can say:

\[
\mu = \frac{d}{t_d E}
\]

Analysis

- Unfortunately it is not quite this simple. Mainly due to carrier diffusion.
- In reality we have to use the minority carrier continuity equation:

\[
\frac{\partial (\Delta n)}{\partial t} = D_n \frac{\partial^2 (\Delta n)}{\partial x^2} + \mu_n E \frac{\partial (\Delta n)}{\partial x} - \frac{\Delta n}{\tau_n}
\]

- This is for a p-type semiconductor, so that electrons are minority carriers.
Analysis

• A solution to this equation is given by:

\[
\Delta n(x, t) = \frac{N}{\sqrt{4\pi D_n t}} \exp \left[ -\frac{(x - \mu_n Et)^2}{4D_n t} - \frac{t}{\tau_n} \right]
\]

• \(N\) is the electron density at \(t = 0\).

Analysis

• Thus, the injected electron packet broadens and decreases in amplitude as it drifts in the field:

\[
\Delta n(x, t)
\]

• In addition to estimating the mobility, the time-of-flight method also offers an approach for estimating the diffusion constant \(D_n\), and lifetime, \(\tau_n\).
Diffusion Coefficient

- The diffusion coefficient is given by:

\[ D_n = \frac{(d\Delta t)^2}{16 \ln(2)t_d^3} \]

- Where \( \Delta t \) is the full-width-at-half-maximum (FWHM) of the measured pulse width.

Lifetime

- The lifetime can be estimated from a measurement of two different drift times, \( t_{d1} \) and \( t_{d2} \), measured under two different drift voltages, \( V_{d1} \) and \( V_{d2} \).

- This leads to:

\[ \tau_n = \frac{t_{d2} - t_{d1}}{\ln \left( \frac{V_{01}}{V_{02}} \right) - 0.5 \ln \left( \frac{t_{d2}}{t_{d1}} \right)} \]

- where \( V_{01} \) and \( V_{02} \) are the corresponding output pulse amplitudes.
Time-of-Flight Drift Mobility

- There are many variations on the time-of-flight measurement.
- Carrier injection or detection can be accomplished optically, using an electron beam, or using a microwave circuit.
- Different kinds of device structures may also be employed.
- However many in the community consider it an unreliable technique. There are many complications due to injection barriers etc.
- Measured mobilities also tend to be $\ll \mu_{FET}, \mu_H$.

Next Time

- Review Lecture for Midterm.

Exam structure / regulations.

Example questions.