Lecture 11
Midterm Review Lecture

Announcements

Homework 3/6:
• Is online now.
• Due Monday 14th May at 10:00am.
Lecture 11

- Exam Format.
- Units.
- Example Questions.
  - Note: this is not an exhaustive list of questions. These are here to give you practice and to inform as to what type of question you can expect to see.
  - Do not use this lecture alone to prepare for the exam!
Front Page

ChE 613/ECE 613
OREGON STATE UNIVERSITY
Midterm Examination May 4th 2018
Electronic Materials and Characterization

Answer 2 out of 3 questions. If more than 2 questions are answered, the 2 questions with the highest marks will be considered.

A maximum of 10 marks will be awarded for each question. The mark assigned to each part of the question is indicated by a number in square brackets at the end of the sub-question, e.g. [3 marks].

You have 45 minutes to complete this exam; please pace yourself accordingly.

No notes of any kind are allowed. You may use calculators for this exam.

Questions Start on page 6.

Useful Constants:

Details

• Friday May 4th at 10:00am in STAG113
• Exam will last 45 minutes.
  • The exam will start exactly at 10:00am!
• Closed book and closed notes.
  • You can, and are expected to, use a calculator.
• Choose 2 out of 3 questions.
  • If you answer 3 I will take best 2 scores.
• It will contribute 25% of overall grade for class.
• The exam will material covered in lectures 2-10 (inclusive).
• There will be 10 marks per question. 20 total.
  • Each mark = 5% of exam grade.
Details

- All constants will be provided.
- All relevant formulae will also be provided.
- Parameters will be labeled as clearly as possible.

Units
Unit Conversions

- It is worth spending a few slides on units.
- In semiconductor characterization we tend to use often non-SI units as standard parameters.
- In particular, the use of cm is common over m.
- For example:
  - Concentration, \( n, p \sim \text{cm}^{-3} \) rather than \( \text{m}^{-3} \).
  - Mobility, \( \mu_e, \mu_h \sim \text{cm}^2/\text{Vs} \) rather than \( \text{m}^2/\text{Vs} \).
- You are expected to know how to convert between units without notes / references.
- This is a useful skill generally.

Unit Conversions

- In one dimension it is straightforward:
  
  \[
  1 \text{ m} = 100 \text{ cm} \\
  1 \text{ cm} = 0.01 \text{ m}
  \]

- It can be easier to go from a unit to meters before going to cm:
  
  \[
  1 \text{ \( \mu \)m} = 10^{-6} \text{ m} \quad \text{and} \quad 10^{-6} \text{ m} = 10^{-4} \text{ cm} \\
  1 \text{ km} = 10^{3} \text{ m} \quad \text{and} \quad 10^{3} \text{ m} = 10^{5} \text{ cm}
  \]
Inverse Dimensions

- For inverse dimensions we go the other way:
  
  \[
  1 \text{ m} = 100 \text{ cm} \quad \Rightarrow \quad 1 \text{ m}^{-1} = 0.01 \text{ cm}^{-1}
  \]
  
  \[
  1 \text{ cm} = 0.01 \text{ m} \quad \Rightarrow \quad 1 \text{ cm}^{-1} = 100 \text{ m}^{-1}
  \]

- E.g. If you can have a line that is one meter long, it can accommodate 100 items in this meter. I.e. \(100 \text{ m}^{-1}\). If you shorten the length to 1 cm, it can now only hold 1 item. I.e. \(1 \text{ cm}^{-1}\).

Inverse Dimensions

- For inverse dimensions we go the other way:
  
  \[
  1 \text{ m} = 100 \text{ cm} \quad \Rightarrow \quad 1 \text{ m}^{-1} = 0.01 \text{ cm}^{-1}
  \]
  
  \[
  1 \text{ cm} = 0.01 \text{ m} \quad \Rightarrow \quad 1 \text{ cm}^{-1} = 100 \text{ m}^{-1}
  \]

- Again, it is often easier to convert to meters first.
  
  \[
  1 \text{ nm}^{-1} = 10^{9} \text{ m}^{-1} \quad \Rightarrow \quad 10^{9} \text{ m}^{-1} = 10^{7} \text{ cm}^{-1}
  \]
  
  \[
  1 \text{ km}^{-1} = 10^{-3} \text{ m}^{-1} \quad \Rightarrow \quad 10^{-3} \text{ m}^{-1} = 10^{-5} \text{ cm}^{-1}
  \]
**Area**

- When talking about a length dimension squared, you apply the original conversion twice:
  
  \[
  1 \text{ m} = 100 \text{ cm} \quad \Rightarrow \quad 1 \text{ m}^2 = 100 \times 100 \text{ cm}^2 = 10^4 \text{ cm}^2
  \]
  
  \[
  1 \text{ cm} = 0.01 \text{ m} \quad \Rightarrow \quad 1 \text{ cm}^2 = 0.01 \times 0.01 \text{ m}^1 = 10^{-4} \text{ m}^2
  \]

- This should be self-evident from geometry:

- Again it is often easier to convert to meters first:
  
  \[
  1 \text{ nm}^2 = 10^{-18} \text{ m}^2 \quad \Rightarrow \quad 10^{-18} \text{ m}^2 = 10^{-14} \text{ m}^2
  \]
  
  \[
  1 \text{ km}^2 = 10^6 \text{ m}^2 \quad \Rightarrow \quad 10^6 \text{ m}^2 = 10^{10} \text{ cm}^2
  \]
Inverse Area

- For inverse area we apply the inverse conversion twice:
  \[ 1 \text{ m} = 100 \text{ cm} \quad \Rightarrow \quad 1 \text{ m}^{-1} = 0.01 \text{ cm}^{-1} \quad \Rightarrow \quad 1 \text{ m}^{-2} = 10^{-4} \text{ cm}^{-2} \]
  \[ 1 \text{ cm} = 0.01 \text{ m} \quad \Rightarrow \quad 1 \text{ cm}^{-1} = 100 \text{ cm}^{-1} \quad \Rightarrow \quad 1 \text{ cm}^{-2} = 10^4 \text{ m}^{-2} \]

- Again it is often easier to convert to meters first.
  \[ 1 \text{ nm}^{-2} = 10^{18} \text{ m}^{-2} \quad \Rightarrow \quad 10^{18} \text{ m}^{-2} = 10^{14} \text{ cm}^{-2} \]
  \[ 1 \text{ km}^{-2} = 10^{-6} \text{ m}^{-2} \quad \Rightarrow \quad 10^{-6} \text{ m}^{-2} = 10^{-10} \text{ cm}^{-2} \]

Volume

- For volume you apply the conversion 3 times
  \[ 1 \text{ m} = 100 \text{ cm} \quad \Rightarrow \quad 1 \text{ m}^2 = 10^4 \text{ cm}^2 \quad \Rightarrow \quad 1 \text{ m}^3 = 10^6 \text{ cm}^3 \]
  \[ 1 \text{ cm} = 0.01 \text{ m} \quad \Rightarrow \quad 1 \text{ cm}^2 = 10^{-4} \text{ m}^2 \quad \Rightarrow \quad 1 \text{ cm}^3 = 10^{-6} \text{ m}^3 \]

- Again it is often easier to convert to meters first.
  \[ 1 \text{ nm}^3 = 10^{-27} \text{ m}^3 \quad \Rightarrow \quad 10^{-27} \text{ m}^3 = 10^{-21} \text{ cm}^3 \]
  \[ 1 \text{ km}^3 = 10^9 \text{ m}^3 \quad \Rightarrow \quad 10^9 \text{ m}^3 = 10^{15} \text{ cm}^3 \]
Inverse Volume

• Finally, for inverse volume you apply the conversion in reverse 3 times

\[ 1 \text{ m} = 100 \text{ cm} \quad \rightarrow \quad 1 \text{ m}^{-2} = 10^{-4} \text{ cm}^{-2} \quad \rightarrow \quad 1 \text{ m}^{-3} = 10^{-6} \text{ cm}^{-3} \]

\[ 1 \text{ cm} = 0.01 \text{ m} \quad \rightarrow \quad 1 \text{ cm}^{-2} = 10^{-4} \text{ m}^{-2} \quad \rightarrow \quad 1 \text{ cm}^{-3} = 10^{-6} \text{ m}^{-3} \]

• Again it is often easier to convert to meters first.

\[ 1 \text{ nm}^{-3} = 10^{27} \text{ m}^{-3} \quad \rightarrow \quad 10^{27} \text{ m}^{-3} = 10^{21} \text{ cm}^{-3} \]

\[ 1 \text{ km}^{-3} = 10^{-9} \text{ m}^{-3} \quad \rightarrow \quad 10^{-9} \text{ m}^{-3} = 10^{-15} \text{ cm}^{-3} \]

Non-SI Units in Equations

• If all parameters in an equation are in SI units you are guaranteed to get the correct answer.
  • You may have to convert input parameters to SI, depending on how they are provided.
  • You also may have to convert the final answer from SI, if the question asks for a non-SI final unit.

• However sometimes it is more convenient to just use the non SI unit.
• For example if input parameters are in cm, and final answer is in Ω cm.
Non-SI Units in Equations

- For example, the resistivity in terms of resistance and dimensions:
  \[ \rho = \frac{RA}{L} \]

- If we are told:
  - \( R = 1\Omega \).
  - \( A = 1\text{cm}^2 \).
  - \( L = 1\text{cm} \).
  - We want answer in \( \Omega\text{cm} \)

- So we can just use cm directly.

Non-SI Units in Equations

- However consider this example of photon energy:
  \[ E = \frac{hc}{\lambda} \]

- If for example we are told:
  - \( \lambda = 1\mu\text{m} \).
  - We want the energy in J.

  Unit mismatch
Non-SI Units in Equations

• However consider this example of photon energy:

\[ E = \frac{hc}{\lambda} \]

• If for example we are told:
  • \( \lambda = 1 \mu m. \)
  • We want the energy in J.

\[ E = \frac{hc}{\lambda} \]

\[ 3.0 \times 10^{14} \mu m/s \]

Lecture 2
Example 1

• Consider the following thin film of indium tin oxide (ITO):

![Diagram of thin film with dimensions 10 μm x 100 nm x 100 μm]

• If we measure a resistance between these two electrodes of 7Ω, what is the conductivity of the ITO film?

Example 1

• Need to determine which direction current flows:

![Diagram with current direction indicated]

• So we can consider the “resistor” to have the following dimensions:
Example 1

• Hence in this case the area is a very thin slab:

\[
A = 100 \times 10^{-6} \times 100 \times 10^{-9} = 1 \times 10^{-11} \text{m}^2
\]

\[
L = 1 \times 10^{-5} \text{m}
\]

Example 1

• So the dimensions are:

\[
\rho = \frac{RA}{L}
\]

\[
\rho = \frac{7 \times 10^{-11}}{10^{-5}} = 7 \times 10^{-6} \Omega \text{m}
\]

\[
= 7 \times 10^{-4} \Omega \text{cm}
\]
Example 2

• Consider intrinsic (undoped) silicon.
• If we approximate the hole and electron densities are room temperature as:
  • \( n_i = p_i = 10^{10} \text{ cm}^{-3} \).
• And the room temperature carrier mobilities as:
  • \( \mu_e = 1400 \text{ cm}^2/\text{Vs} \).
  • \( \mu_h = 500 \text{ cm}^2/\text{Vs} \).
• What is the room temperature conductivity of intrinsic silicon (in the dark)?

Example 2

• Use our equation for conductivity (would be given):

\[
\sigma = q(\mu_e n + \mu_h p)
\]

• Put in the numbers (work in cm):

\[
\sigma = 1.6 \times 10^{-19}(1400 \times 10^{10} + 500 \times 10^{10})
\]

\[
\sigma = 1.6 \times 10^{-19} \times 1900 \times 10^{10}
\]

\[
\boxed{\sigma = 3.04 \times 10^{-6} \text{ S cm}^{-1}}
\]
Example 1

- Consider a 4-probe measurement on a sample we can approximate as infinitely thick.

- Let:

\[ S_1 = S_2 = S_3 = S \]
Example 1

- If we drive a current of $I = 1\text{mA}$ across the outer two probes, we measure a voltage drop of 40μV between the inner two.
- If the probe separation is 500 μm, determine the resistivity of the sample. Give your answer in Ωcm.
- You would be given:

$$\rho = 2\pi S \frac{V}{I}$$

Example 1

$$\rho = 2\pi S \frac{V}{I}$$

- So we can just put in numbers.
- We can work in m to start:

$$\rho = 2\pi \times 500 \times 10^{-6} \times \frac{40 \times 10^{-6}}{10^{-3}}$$

$$\rho = 0.0031 \ \Omega m$$

$$\rho = 0.31 \ \Omega cm$$
Example 2

• For a sample of finite thickness the resistivity is given by:

\[
\rho = 2\pi SF \frac{V}{I}
\]

• Where:
  • \(\rho\) is the resistivity of the sample.
  • \(S\) is the sample thickness.
  • \(V\) is the voltage.
  • \(I\) is the current.
  • \(F\) is a correction factor.

• List 3 things which affect \(F\).

Example 2

• List 3 things which affect \(F\).

\[
\rho = 2\pi SF \frac{V}{I}
\]

• \(F\) is comprised of several factors.

\[F = F_1 F_2 F_3\]

• \(F_1\) is associated with the sample thickness.
• \(F_2\) is associated with the lateral sample dimensions.
• \(F_3\) is associated the probe locations relative to the sample edges.
Example 3

- Below is an equation for sheet resistance.

\[ R_{sh} = \frac{\pi V}{\ln 2 I} = 4.532 \frac{V}{I} \]

- What units are conventional for sheet resistance?

\[ \Omega/\square \]

- To indicate this is sheet resistance, it is normally expressed in ohms per square (\(\Omega/\square\)), but is dimensionally a resistance.

Example 4

- Consider a measurement on sample of arbitrary geometry:

- If we apply a voltage of 100 mV between probes 3 and 4, we measure a current of 40 mA between probes 1 and 2.
- If we apply a voltage of 100 mV between probes 1 and 4, we measure a current of 20 mA between probes 2 and 3.
Example 4

• If this sample is assumed to be uniformly 500 μm thick, homogeneous and isotropic, what is its resistivity?

• We would be given the following equation for an arbitrary geometry:

\[ \rho = \frac{\pi}{\ln 2} t \left( \frac{R_{12,34} + R_{23,41}}{2} \right) F_v \]

Sample thickness

\[ R_{12,34} = \frac{V_{34}}{I_{12}} \]

\[ R_{23,41} = \frac{V_{41}}{I_{23}} \]

Correction factor associated with arbitrary geometry

Example 4

• Evaluate resistances:

\[ R_{12,34} = \frac{V_{34}}{I_{12}} = \frac{0.1}{0.04} \]

\[ R_{12,34} = 2.5 \Omega \]

\[ R_{23,41} = \frac{V_{41}}{I_{23}} = \frac{0.1}{0.02} \]

\[ R_{23,41} = 5 \Omega \]

• Evaluate ratio of resistances:

\[ R_r = \frac{R_{12,34}}{R_{23,41}} \]

\[ R_r = \frac{2.5}{5} \]

\[ R_r = 0.5 \]
Example 4

- Evaluate correction factor.
- The correction factor is given by:
  \[
  \frac{R_r - 1}{R_r + 1} = \frac{F_v}{\ln 2} \arccosh \left( \frac{\exp[\ln(2)/F_v]}{2} \right)
  \]
  \[
  R_r = \frac{R_{12,34}}{R_{23,41}}
  \]
- You would not be expected to calculate something numerically.
- So, you would be told \( F_v \) for a given \( R_r \)
- Or a graph would be provided.

---

Example 4

- E.g. would be told for our value of \( R_r \) that \( F_v = 0.96 \).
- Then we can just put numbers in:
  \[
  \rho = \frac{\pi}{\ln 2} t \left( \frac{R_{12,34} + R_{23,41}}{2} \right) F_v
  \]
  \[
  \rho = \frac{\pi}{\ln 2} \times 500 \times 10^{-6} \times \frac{(2.5 + 5.0)}{2} \times 0.96
  \]
  \[
  \rho = 0.00816 \, \Omega \text{m}
  \]
  \[
  \rho = 0.816 \, \Omega \text{cm}
  \]
Example 5

• The correction factor for a van der Pauw measurement is given by:

\[
\frac{R_r - 1}{R_r + 1} = \frac{F_v}{\ln 2} \text{arccosh} \left( \frac{\exp[\ln(2)/F_v]}{2} \right) \quad R_r = \frac{R_{12,34}}{R_{23,41}}
\]

• Using this equation, explain why we often use a symmetric geometry (i.e. a geometry with equal distances) for a van der Pauw measurement.

• If distances are equal we can say:

\[
R_{12,34} = R_{23,41} \quad \Rightarrow \quad R_r = 1
\]

Example 5

• If \( R_r = 1 \):

\[
\frac{1 - 1}{2 + 1} = 0 = \frac{F_v}{\ln 2} \text{arccosh} \left( \frac{\exp[\ln(2)/F_v]}{2} \right)
\]

\[
\text{arccosh} \left( \frac{\exp[\ln(2)/F_v]}{2} \right) = 0 \quad \Rightarrow \quad \frac{\exp[\ln(2)/F_v]}{2} = \cosh(0)
\]

\[
\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \Rightarrow \quad \cosh(0) = \frac{e^0 + e^0}{2} \quad \Rightarrow \quad \cosh(0) = 1
\]
Example 5

\[ \cosh(0) = \frac{\exp[\ln(2)/F_v]}{2} = 1 \]

\[ \exp[\ln(2)/F_v] = 2 \]

- Take logs:
  \[ \frac{\ln(2)}{F_v} = \ln(2) \quad \Rightarrow \quad F_v = \frac{\ln(2)}{\ln(2)} = 1 \]

- I.e. if \( R_{12,34} = R_{23,41} \), \( F_v = 1 \).
- Hence if the path lengths are equal, the analysis becomes much more straightforward.

Example 6

- Consider the following hot-probe measurement.

- If we are measuring a p-type sample, what sign would the voltage \( V_{12} = V_1 - V_2 \) in such a measurement?
Example 6

- In a hot-probe experiment, a concentration gradient will exist between the hot and cold regions:

![Diagram showing concentration gradient with low hole density on the left and high hole density on the right.]

- If holes are the majority carrier, the potential at probe 1 will be lower than at probe 2.
- Hence the potential between probe 1 and 2 ($V_{12} = V_1 - V_2$) will be negative.

Lecture 4
Example 1

- Consider a silicon sample, doped with $10^{20}$ cm$^{-3}$ donors.
- At a temperature of $T = 77$K would we expect thermionic emission, field emission, or thermionic-field emission to dominate?
- Assume:
  - $\varepsilon_r = 11.7$ (silicon).
  - $m_{tun}^* = 0.3 m_e$.

Example 1

- To solve this we will have to evaluate the characteristic energy.

$$E_{00} = \frac{eh}{4\pi} \sqrt{\frac{N_D}{\varepsilon_0 \varepsilon_r m_{tun}^*}}$$

- If $k_B T \gg E_{00}$ Thermionic Emission (TE) dominates.
- If $k_B T \approx E_{00}$ Thermionic-Field Emission (TFE) dominates.
- If $k_B T \ll E_{00}$ Field Emission (FE) dominates.
- So basically we just calculate $E_{00}$ and compare it $k_B T$. 
Example 1

- Put numbers in:

\[
E_{00} = \frac{eh}{4\pi} \left( \frac{N_D}{\varepsilon_0 \varepsilon_r m_{\text{eun}}} \right)
\]

- Vacuum permittivity has units of Fm\(^{-1}\), so we must use meters!

\[
N_D = 10^{20} \text{ cm}^{-3} \quad \rightarrow \quad N_D = 10^{26} \text{ m}^{-3}
\]

\[
E_{00} = \frac{1.60 \times 10^{-19} \times 6.63 \times 10^{-34}}{4\pi} \sqrt{\frac{10^{26}}{8.85 \times 10^{-12} \times 11.7 \times 0.3 \times 9.11 \times 10^{-31}}}
\]

Example 1

- Put numbers in:

\[
E_{00} = 1.59 \times 10^{-20} \text{ J}
\]

- Now evaluate \( k_B T \):

\[
k_B T = 1.38 \times 10^{-23} \times 100
\]

\[
k_B T = 1.38 \times 10^{-21} \text{ J}
\]

- So:

\[
E_{00} \gg k_B T
\]

- Field Emission (FE) dominates.
Example 2

- Consider a sample with a circular contact of radius $r = 200 \mu m$, a resistivity of $\rho = 3 \times 10^5 \Omega cm$, and a thickness $t = 2 mm$.

- **Determine the spreading resistance ($R_{SP}$) at this contact.**

- You would be given:

$$R_{SP} = \frac{\rho}{2\pi r} \arctan \left( \frac{2t}{r} \right)$$

- So this would just be an exercise in entering numbers.

Example 2

- Just enter numbers, all in m:

$$R_{SP} = \frac{\rho}{2\pi r} \arctan \left( \frac{2t}{r} \right)$$

$$R_{SP} = 2.4 \times 10^6 \arctan(20)$$

$$R_{SP} = 3.6 \times 10^6 \Omega$$
Example 3

- We carry out a current-voltage measurement on structure to the right:

- If we are told that:
  - $I = 10$ mA.
  - $L = 10$ μm.
  - $Z = 50$ μm.
  - $Z = 50$ μm.
  - $R_{sh} = 10$ Ω/□.
  - $\rho_c = 10^{-6}$ Ωcm$^2$.

- What is the voltage under the middle of the electrode?

Example 3

- In this case we would be given the expression for voltage under the contact:

$$V(x) = I \sqrt{R_{sh} \rho_c} \frac{\cosh \left[ \frac{L - x}{L_T} \right]}{Z} \frac{\sinh \left[ \frac{L}{L_T} \right]}{\sinh \left[ \frac{L}{L_T} \right]}$$

- Under the middle of the contact $x = L/2$.
- So we need to evaluate:

$$V(L/2)$$
Example 3

• First evaluate transfer length:

\[ L_T = \frac{\sqrt{\rho_c}}{R_{sh}} \]

• \( \rho_c = 10^{-6} \text{ Ωcm}^2 \rightarrow \rho_c = 10^{-10} \text{ Ωm}^2 \)

\[ L_T = \frac{10^{-10}}{10} = 3.16 \times 10^{-6} \text{ m} \]

Example 3

• Now evaluate \( V \):

\[ V(x) = I \sqrt{R_{sh}\rho_c} \frac{\cosh \left[ \frac{L - x}{L_T} \right]}{Z \sinh \left[ \frac{L}{L_T} \right]} \]

\[ V(x = L/2) = I \sqrt{R_{sh}\rho_c} \frac{\cosh \left[ \frac{L}{2L_T} \right]}{Z \sinh \left[ \frac{L}{L_T} \right]} \]

\[ V(L/2) = I \sqrt{R_{sh}\rho_c} \frac{\cosh \left[ \frac{2L - L}{2L_T} \right]}{Z \sinh \left[ \frac{L}{L_T} \right]} \]
Example 3

\[ V(L/2) = \frac{I}{Z} \sqrt{\frac{\sqrt{R_s h} \rho_c}{2 L_T}} \cos h \left[ \frac{L}{Z L_T} \right] \sin h \left[ \frac{L}{L_T} \right] \]

- Now just enter numbers (use SI):

\[ V(L/2) = \frac{10 \times 10^{-3} \times \sqrt{10 \times 10^{-10}} \cosh \left[ \frac{10 \times 10^{-6}}{2 \times 3.16 \times 10^{-6}} \right]}{50 \times 10^{-6}} \sinh \left[ \frac{10 \times 10^{-6}}{3.16 \times 10^{-6}} \right] \]

\[ V(L/2) = \frac{10^{-2} \times \sqrt{10^{-9}} \cosh[1.58]}{5 \times 10^{-5}} \sinh[3.16] \]

\[ V(L/2) = 6.32 \times 10^{-3} \cosh[1.58] \sinh[3.16] \]

Example 3

- Note:

\[ \cosh \theta = \frac{e^{\theta} + e^{-\theta}}{2} \quad \sinh \theta = \frac{e^{\theta} - e^{-\theta}}{2} \]

\[ V(L/2) = 6.32 \times 10^{-3} \times 0.214 \]

\[ V(L/2) = 1.36 \times 10^{-3} \ V \]
Example 4

- Consider the following transmission line measurement:

- We are told that $W = Z$.
- If we plot total resistance measured ($R_T$) as a function of electrode separation ($d$), we observe an extrapolated y-axis intercept of 50 Ω. What is the contact resistance?

Example 4

- We would be given the equation for the total resistance between any two contacts:

$$R_T = \frac{R_{sh}d_i}{Z} + 2R_c$$

- We can identify this as our equation for a straight line.
  - Where $2R_c$ is the y-axis intercept.
  - I.e.

$$2R_c = 50Ω$$

$$R_c = 25Ω$$
Example 1

- A capacitance-voltage measured is carried out on a sample.
- $1/C^2$ is plotted against magnitude of voltage.
- If the gradient of this plot is $10^{21} \, \text{F}^{-2} \, \text{V}^{-1}$, determine the doping density in this sample. Give you answer in $\text{cm}^{-3}$.
- We are told the sample is a square pad of $A = 100 \mu\text{m} \times 100 \mu\text{m}$, the relative permittivity is $\varepsilon_r = 12$. 
Example 1

• We would be given:

\[ N_A(W) = \frac{2}{q A^2 \varepsilon_0 \varepsilon_r \frac{d}{dV} \left( \frac{1}{C^2} \right)} \]

• We would be able to identify our gradient:

\[ \frac{d}{dV} \left( \frac{1}{C^2} \right) = 10^{21} \text{ F}^{-2} \text{V}^{-1} \]

• The remainder of the question is just entering numbers.

Example 1

\[ N_A(W) = \frac{2}{q A^2 \varepsilon_0 \varepsilon_r \frac{d}{dV} \left( \frac{1}{C^2} \right)} \]

• First get the area:

\[ A = 100 \times 10^{-6} \times 100 \times 10^{-6} = 10^{-8} \text{ m}^2 \]

\[ A^2 = 10^{-8} \times 10^{-8} = 10^{-16} \text{ m}^2 \]

• Now enter numbers:

\[ N_A(W) = \frac{2}{1.60 \times 10^{-19} \times 10^{-16} \times 8.85 \times 10^{-12} \times 12 \times 10^{21}} \]
Example 1

\[ N_A(W) = \frac{2}{1.60 \times 10^{-19} \times 10^{-16} \times 8.85 \times 10^{-12} \times 12 \times 10^{21}} \]

\[ N_A(W) = 1.18 \times 10^{24} \text{ m}^{-3} \]

\[ N_A(W) = 1.18 \times 10^{18} \text{ cm}^{-3} \]

Example 2

- The equation for an ideal diode is given below:

\[ I(V) = I_S \left[ \exp \left( \frac{qV}{n k_B T} \right) - 1 \right] \]

- Where:
  - \( q \) = Fundamental unit of charge.
  - \( n \) = Ideality factor (~1).
  - \( k_B \) = Boltzmann Constant.
  - \( T \) = Temperature.
  - \( I_S \) = Reverse saturation current.
Example 2

- What is meant by reverse saturation current ($I_S$)?
- This is the current that flows when a large reverse bias is applied.

$$I(V) = I_S \left[ \exp \left( \frac{qV}{nk_B T} \right) - 1 \right] \Rightarrow I(V \to -\infty) = I_S$$

Example 3

- The current flowing in a diode ($I$), with an ideality factor $n = 1$, was measured at a constant voltage of $V = 0.2\text{V}$, and as a function of temperature ($T$).
- The data was plotted as $I/T^2$ vs $1000/T$ on a logarithmic y-axis scale.
- We are told the gradient (on log$_{10}$ scale) of this plot is $-0.25 \text{ AK}^{-2}$.
- From this figure, determine the injection barrier ($\phi_B$). Give your answer in V.
Example 3

- We would be given the following approximation:

\[
I = AA^*T^2 \exp \left( \frac{q(V/n - \phi_B)}{k_B T} \right)
\]

- And we would be given the Richardson constant:
  - \( A^* = 32 \text{ Acm}^{-2}\text{K}^{-2} \).
  - Or we would be given the effective mass, and be expected to calculate \( A^* \).

\[
A^* = \frac{4\pi q k_B^2 m^*}{h^3}
\]

Example 3

- First we need to re-arrange the equation for \( I \) so that it matches our provided plot.

\[
\frac{I}{AA^*T^2} = \exp \left( \frac{q(V/n - \phi_B)}{k_B T} \right)
\]

- Take (natural) logs:

\[
\ln \left( \frac{I}{AA^*T^2} \right) = \frac{q(V/n - \phi_B)}{k_B T}
\]
Example 3

\[
\ln\left(\frac{I}{AA^*T^2}\right) = \frac{q(V/n - \phi_B)}{k_B T}
\]

- Re-arrange:

\[
\ln\left(\frac{I}{T^2}\right) = \ln(AA^*) - \frac{q(\phi_B - \frac{V}{n})}{k_B T}
\]

- The y-axis is plotted on a log10 scale. So we need to use (would be given):

\[
\log_a(x) = \frac{\ln(x)}{\ln(a)} \quad \text{log}_{10}(x) = \frac{\ln(x)}{\ln(10)}
\]

\[
\ln(x) = 2.3\log_{10}(x)
\]

Example 3

\[
\ln(x) = 2.3\log_{10}(x) \quad \ln\left(\frac{I}{T^2}\right) = \ln(AA^*) - \frac{q(\phi_B - \frac{V}{n})}{k_B T}
\]

\[
\ln\left(\frac{I}{T^2}\right) = 2.3\log_{10}\left(\frac{I}{T^2}\right) = \ln(AA^*) - \frac{q(\phi_B - \frac{V}{n})}{k_B T}
\]

\[
\log_{10}\left(\frac{I}{T^2}\right) = \frac{\ln(AA^*)}{2.3} - \frac{q(\phi_B - \frac{V}{n})}{2.3k_B T}
\]

- Hence the y-axis is now correct.

\[
y = mx + c
\]
Example 3

\[ \log_{10} \left( \frac{I}{T^2} \right) = \frac{\ln(\frac{AA'}{2.3})}{2.3} - \frac{q \left( \frac{\phi_B}{n} - \frac{V}{2300k_B} \right)}{2300k_B} \]

\[ y = mx + c \]

- The x-axis is: \( 1000/T \)

\[ x = \frac{1000}{T} \]

\[ \frac{x}{1000} = \frac{1}{T} \]

\[ \log_{10} \left( \frac{I}{T^2} \right) = \frac{\ln \left( \frac{AA'}{2.3} \right)}{2.3} - \frac{q \left( \frac{\phi_B}{n} - \frac{V}{2300k_B} \right)}{2300k_B}x \]

Example 3

\[ \log_{10} \left( \frac{I}{T^2} \right) = \frac{\ln(\frac{AA'}{2.3})}{2.3} - \frac{q \left( \frac{\phi_B}{n} - \frac{V}{2300k_B} \right)}{2300k_B}x \]

- So we can identify the gradient as:

\[ m = -\frac{q \left( \frac{\phi_B}{n} - \frac{V}{n} \right)}{2300k_B} \]

- Now re-arrange:

\[ \phi_B = \frac{V}{n} - \frac{m2300k_B}{q} \]
Example 3

\[ \phi_B = \frac{V}{n} - \frac{m2300k_B}{q} \]

• We are given \( V = 0.2, \ n = 1, \ m = -0.25. \)

\[ \phi_B = \frac{0.2}{1} - \frac{-0.25 \times 2300 \times 1.38 \times 10^{-23}}{1.60 \times 10^{-19}} \]

\[ \phi_B = 0.2 + \frac{7.935 \times 10^{-21}}{1.60 \times 10^{-19}} \]

\[ \phi_B = 0.2 + 0.05 \]

\[ \phi_B = 0.25 \text{ V} \]

Lecture 6
Example 1

- Give three ways in which we can define a deep level impurity.
  - **Definition 1:** A level is deep if its binding energy is greater than 0.1 eV.
  - **Definition 2:** A level is deep if it is non-effective-mass-like.
  - **Definition 3:** A level is deep if it is important in non-radiative recombination, i.e., Shockley-Read-Hall recombination.

Example 2

- The rate of electron capture in a semiconductor trap is given by:
  \[
  r_a = \sigma_n v_{th} n N_T (1 - f_T)
  \]

  - Where:
    - \( r_a \) = Rate of electron capture.
    - \( \sigma_n \) = Electron capture cross-section (e.g. cm\(^2\)).
    - \( v_{th} \) = Electron thermal velocity.
    - \( n \) = Delocalized electron density in conduction band.
    - \( N_T (1 - f_T) \) = Number density of empty traps.
Example 2

- The rate of electron emission is given by:

\[ r_b = e_n N_T f_T \]

- Where:
  - \( r_b \) = Rate of electron emission.
  - \( e_n \) = Proportionality constant equal to the rate at which a single electron in an occupied trap will jump into the conduction band.
  - \( N_T f_T \) = Number density of filled traps.

Example 2

- If we can describe trap occupancy by Fermi-Dirac statistics, derive an expression for electron emission rate \( (e_n) \) under steady state conditions, in terms of the capture cross section \( (\sigma_n) \), thermal velocity \( (v_{th}) \), number density of conduction band states \( (N_c) \), trap energy \( (E_T) \), conduction band energy \( (E_C) \), and temperature \( (T) \).

  - We would be given:

\[
    f_T = \frac{1}{1 + \exp \left( \frac{E_T - E_F}{k_b T} \right)} \quad n = N_c \exp \left( - \frac{E_C - E_F}{k_b T} \right)
\]
Example 2

- First, since we are in steady state, we can equate the rates:

\[ r_a = r_b \]

\[ \sigma_n v_{th} n N_T (1 - f_T) = e_n N_T f_T \]

\[ e_n = \sigma_n v_{th} n \frac{1 - f_T}{f_T} \]

- We can also re-arrange the Fermi-Dirac equation:

\[ f_T = \frac{1}{1 + \exp \left( \frac{E_T - E_F}{k_b T} \right)} \]

\[ \frac{1 - f_T}{f_T} = \exp \left( \frac{E_T - E_F}{k_b T} \right) \]

- Combine this with our equation for emission rate:

\[ \frac{1 - f_T}{f_T} = \exp \left( \frac{E_T - E_F}{k_b T} \right) \]

\[ e_n = \sigma_n v_{th} n \frac{1 - f_T}{f_T} \]

\[ e_n = \sigma_n v_{th} n \exp \left( \frac{E_T - E_F}{k_b T} \right) \]

- Substitute in our given equation for \( n \):

\[ e_n = \sigma_n v_{th} N_c \exp \left( - \frac{E_C - E_F}{k_b T} \right) \exp \left( \frac{E_T - E_F}{k_b T} \right) \]

\[ e_n = \sigma_n v_{th} N_c \exp \left( - \frac{E_C - E_T}{k_b T} \right) \]
Example 1

• Briefly describe the steps involved in carrying out a deep level transient spectroscopy (DLTS) experiment.

  1. Create a space charge region by applying a DC reverse bias across your measurement device.
  2. Apply a repetitive voltage pulse in which electrons or holes are captured by deep levels.
  3. When the pulse is terminated, the deep levels will attempt to reach their equilibrium occupancy by thermal emission.
Example 1

4. The return to equilibrium can be monitored by measuring capacitance (or conductance or current or charge).

Example 2

- When carrying out a deep level transient spectroscopy (DLTS) experiment, describe with diagrams what is meant by the parameter $T_{\text{max}}$.
- In a DLTS measurement we would measure capacitance as a function of time, at a range of temperatures. As shown to the right.
- The decay rate will depend on temperature.
Example 2

- We define two samples times: $t_1$ and $t_2$, at which we consider the capacitance.
- We can then define the difference in capacitance sampled at these two times as:

$$\delta C = C(t_1) - C(t_2)$$

- When we plot $\delta C$ as a function of temperature, we will observe a peak $\delta C$ at some value of $T$. See figure to the right.
- We describe this maximum change in capacitance as $\delta C_{\text{max}}$.
- We define the temperature at which $\delta C_{\text{max}}$ is measured as $T_{\text{max}}$. 
Example 3

• When carrying out a deep level transient spectroscopy (DLTS) experiment it turns out that a certain $T_{max}$ occurs when we define the sample times $t_1 = 5$ ms and $t_2 = 20$ ms.

• Using this information, determine the emission lifetime for electrons at $T_{max}$.

• We would be given:

\[
\frac{1}{e_n(T_{\text{max}})} = \tau_{n,\text{max}} = \frac{t_2 - t_1}{\ln\left(\frac{t_2}{t_1}\right)}
\]

Example 3

• Just enter numbers:

\[
\tau_{n,\text{max}} = \frac{0.020 - 0.005}{\ln\left(\frac{0.020}{0.005}\right)}
\]

\[
\tau_{n,\text{max}} = \frac{0.015}{\ln(4)}
\]

\[
\tau_{n,\text{max}} = 2.89 \text{ ms}
\]
Example 4

- We measure the electron emission probability \( e_n \) as a function of temperature.
- If we plot \( \log(e_n) \) versus \( 1/T \) we would get something like the following:

\[
\log(e_n) \quad \text{versus} \quad \frac{1000}{T}
\]

Example 4

- If the gradient of this plot was found to be 2.5, determine the energy offset between the trap and the conduction band. Give your answer in eV
- You would be given:

\[
e_n = \sigma_n v_{th} N_c \exp \left( - \frac{E_C - E_T}{k_B T} \right)
\]

\[
\frac{e_n}{\sigma_n v_{th} N_c} = \exp \left( - \frac{E_C - E_T}{k_B T} \right)
\]

- Take logs (base-10):

\[
\log_{10} \left[ \frac{e_n}{\sigma_n v_{th} N_c} \right] = \log_{10} \left[ \exp \left( - \frac{E_C - E_T}{k_B T} \right) \right]
\]
Example 4

• Use the following relationship for logarithms of different base (would be given):

\[ \log_a(x) = \frac{\ln(x)}{\ln(a)} \]

\[ \log_{10}(x) = \frac{\ln(x)}{\ln(10)} \approx \frac{\ln(x)}{2.3} \]

\[ \log_{10}\left[ \frac{e_n}{\sigma_n v_{th} N_c} \right] = \frac{1}{2.3} \ln\left[ \exp\left( -\frac{E_C - E_T}{k_b T} \right) \right] \]

\[ \log_{10}\left[ \frac{e_n}{\sigma_n v_{th} N_c} \right] = \frac{E_T - E_C}{2.3 k_b T} \]

\[ \log_{10}[e_n] - \log_{10}[\sigma_n v_{th} N_c] = \frac{E_T - E_C}{2.3 k_b T} \]

Example 4

\[ \log_{10}[e_n] = \frac{E_T - E_C}{2.3 k_b T} + \log_{10}[\sigma_n v_{th} N_c] \]

• We identify this as the equation for a straight line:

\[ y = mx + c \]

• Hence we can say:

\[ y = \log_{10}[e_n] \quad mx = \frac{E_T - E_C}{2.3 k_b T} \]

• If the \( x \)-axis is \( 1000/T \):

\[ mx = \frac{E_T - E_C}{2.3 k_b T} \quad \frac{1000}{T} m = \frac{E_T - E_C}{2.3 k_b T} \quad m = \frac{E_T - E_C}{2300 k_b} \]
Example 4

- We are after the offset in energy:

\[ m = \frac{E_T - E_C}{2300k_B} \]

\[ E_T - E_C = 2300k_Bm \]

\[ E_T - E_C = 2300 \times 1.38 \times 10^{-23} \times 2.5 \]

\[ E_T - E_C = 7.94 \times 10^{-20} \text{ J} \]

\[ E_T - E_C = 0.496 \text{ eV} \]

Lecture 8
Example 1

• Using diagrams, briefly describe the following processes and how energy is conserved in each case:

1). Shockley-Read-Hall recombination.
2). Radiative recombination.
2). Auger recombination.

Example 1

1). Shockley-Read-Hall recombination.
• In Shockley-Read-Hall (SRH) recombination a charge carrier (electron or hole) recombines with a trap.
• Energy is conserved through multiple-phonon emission (MPE).
Example 1

2). Radiative recombination

- In radiative recombination an electron and a hole recombine (annihilate).
- Energy is conserved through the emission of a photon.

Example 1

2). Auger recombination

- In Auger recombination an electron and a hole recombine (annihilate).
- However rather than emit a photon, energy is carried away with via a third carrier.
- This can be an electron or a hole.
Example 2

- If the Shockley-Read-Hall (SRH) lifetime is $\tau_{SRH}$, as given below, show that for a p-type material under low light conditions ($\Delta n \ll p_0$), the lifetime reduces to: $\tau_{SRH}^{(ll)} \approx \tau_n$.

\[
\tau_{SRH} = \frac{\tau_p (n_0 + n_1 + \Delta n) + \tau_n (p_0 + p_1 + \Delta p)}{p_0 + n_0 + \Delta n}
\]

\[
\tau_p = \frac{1}{\sigma_p v_{th} N_T} \quad \tau_n = \frac{1}{\sigma_n v_{th} N_T}
\]

- If the sample is p-type:

\[
\Delta p = 0 \quad n_0 = 0
\]

\[
\tau_{SRH}^{(ll)} = \frac{\tau_p (n_1 + \Delta n) + \tau_n (p_0 + p_1)}{p_0 + \Delta n}
\]

- We are also told that this is low level illumination:

\[
\Delta n \ll p_0 \quad \Delta n \approx 0
\]

\[
\tau_{SRH}^{(ll)} = \frac{\tau_p n_1 + \tau_n (p_0 + p_1)}{p_0}
\]
Example 2

\[ \tau_{SRH}^{(II)} = \frac{\tau_p n_1 + \tau_n (p_0 + p_1)}{p_0} \]

\[ \tau_{SRH}^{(II)} = \frac{n_1}{p_0} \tau_p + \left(1 + \frac{p_1}{p_0}\right) \tau_n \]

- \( p_1 \): concentration of holes in the valence band when the Fermi level is positioned at the trap energy level.
- \( n_1 \): concentration of electrons in the conduction band when the Fermi level is located at the trap energy.

\[ n_1, p_1 \ll n_0, p_0 \]

\[ \tau_{SRH}^{(II)} \approx 0 \times \tau_p + (1 + 0) \tau_n \]

\[ \tau_{SRH}^{(II)} \approx \tau_n \]

Example 3

- The band gap of GaAs is 1.43 eV. Determine the wavelength \((\lambda)\) of a photon emitted by radiative recombination in a GaAs device. Give your answer in nm.

- You would be given:

\[ E = \frac{hc}{\lambda} \quad \Rightarrow \quad \lambda = \frac{hc}{E} \]

- Remember to convert energy to joules:

\[ E = 1.43 \text{ eV} \quad \Rightarrow \quad E = 2.29 \times 10^{-19} \text{ J} \]
Example 3

• Enter values:

\[ \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.29 \times 10^{-19}} \]

\[ \lambda = 8.69 \times 10^{-7} \]

\[ \lambda = 869 \text{nm} \]

Example 4

• Briefly describe the concept of photon recycling.

• Photon recycling is the phenomenon of photons that are emitted by radiative recombination in a semiconductor being re-absorbed and generating more electron hole pairs.
Example 1

• A photoconductance decay (PCD) measurement is carried out on a range of wafers with various thicknesses \(d\). The data is shown below.

• We are told that the surface recombination velocity is very small \(s_r \to 0\).

• From this data, determine the bulk carrier lifetime. Give your answer in ms.
Example 1

- You would be given the following equations:

\[
\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_B} + \frac{1}{\tau_s} \quad \frac{1}{\tau_s} = \frac{1}{D\beta^2} \quad \tan \left( \frac{\beta d}{2} \right) = \frac{s_r}{\beta D}
\]

- If \( s_r \) is very small, we can make the small-angle approximation for a \( \tan \) function:

\[
\tan(\theta) \approx \theta \quad \tan \left( \frac{\beta d}{2} \right) \approx \frac{\beta d}{2} = \frac{s_r}{\beta D} \quad \frac{\beta^2 d}{2} = \frac{s_r}{D}
\]

\[
D\beta^2 = \frac{2s_r}{d} \quad \frac{1}{\tau_s} = \frac{1}{D\beta^2} \quad \tau_s (s_r \to 0) = \frac{d}{2s_r}
\]

Example 1

\[
\tau_s (s_r \to 0) = \frac{d}{2s_r}
\]

- Consider the effective lifetime (due to both surface and bulk recombination):

\[
\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_B} + \frac{1}{\tau_s} \quad \Rightarrow \quad \frac{1}{\tau_{\text{eff}}} (s_r \to 0) = \frac{2s_r}{d} + \frac{1}{\tau_B}
\]

- Identify this as the equation for a straight line:

\[
y = mx + c
\]

- Plotting \( 1/\tau_{\text{eff}} \) vs \( 1/d \) will have a slope of \( 2s_r \) and an intercept of \( 1/\tau_B \).
Example 1

• We were told the intercept was 150 s$^{-1}$.

\[ \frac{1}{\tau_B} = 150 \text{ s}^{-1} \]
\[ \tau_B = 6.66 \text{ ms} \]

Example 2

• A surface photovoltage (SPV) measurement is carried out a semiconducting sample, by adjusting the incident photon flux ($\Phi$) and keeping the measured surface voltage ($V_{spv}$) constant.

• From the data in the figure to the right, determine the diffusion length of carriers in this semiconductor.

\[ \Phi = 2 \times 10^9 \text{ (Photons/cm}^2\text{s)} \]
\[ V_{spv} = 10 \text{ mV, } n_{po} = 10^5 \text{ cm}^{-3} \]
\[ s_1 = 1000 \text{ cm/s, } D_n = 30 \text{ cm}^2/\text{s} \]
\[ R = 0.3 \]
\[ \text{Diffusion Length } = 0.003 \text{ cm} \]
Example 2

• You would be given:

\[ \Phi = C_1 \left( L_n + \frac{1}{\alpha(\lambda)} \right) \]

• The intercept of the \( x \)-axis is when \( \Phi = 0 \).

\[ C_1 \left( L_n + \frac{1}{\alpha(\lambda)} \right) = 0 \]

\[ L_n + \frac{1}{\alpha(\lambda)} = 0 \]

\[ \frac{1}{\alpha(\lambda)} = -L_n \]

• I.e. the intercept is just \(-L_n\).

\[ L_n = 0.003 \text{ cm} \]

Example 3

• In the context of electronic devices (e.g. transistors), what is meant by generation lifetime?

• The generation lifetime is the average time that it takes to thermally generate one electron-hole pair in the space charge region under extraction conditions.
Example 1

• We are told that in a certain material, in a certain direction, at a certain temperature, the mean scattering time is $\langle \tau \rangle = 300$ fs, and the effective mass is $m^* = 0.067m_e$
  • $m_e$ is the rest mass of an electron.
• Determine the charge carrier mobility under these conditions. Give your answer in cm$^2$/Vs.
• You would be given:

$$\mu = \frac{q\langle \tau \rangle}{m^*}$$
Example 1

- Just enter numbers:

$$\mu = \frac{1.60 \times 10^{-19} \times 300 \times 10^{-15}}{0.067 \times 9.11 \times 10^{-31}}$$

$$\mu = 0.787 \text{ m}^2/\text{Vs}$$

$$\mu = 7870 \text{ cm}^2/\text{Vs}$$

Example 2

- Below is a schematic of the band structure in graphene.

- Explain why standard (non-relativistic) semiconductor models cannot be applied to determine the effective mass of carriers in such a system.
Example 2

• You would be given the equation for effective mass:

\[ m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \]

• Here we identify the 2nd derivative of the band energy with respect to momentum:

\[ \frac{d^2E}{dk^2} \]

• Notice, on the previous slide, the energy is linearly proportional to momentum:

\[ E = Ak \]

Example 2

• In this case the first derivative would be:

\[ \frac{dE}{dk} = A \]

• And the second derivative would therefore be:

\[ \frac{d^2E}{dk^2} = 0 \]

• And hence the effective mass would be:

\[ m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \quad m^* \rightarrow \frac{\hbar^2}{0} \quad m^* \rightarrow \infty \]
Example 2

- Hence the effective mass is infinite, or undefined for this band structure.
- This problem is solved by considering the effects of special relativity.

Example 3

- Consider the following Hall measurement.
Example 3

- We are told the sample has the following dimensions:
  - \( W = 1 \) mm.
  - \( L = 5 \) mm.
  - \( d = 1 \) mm.
- We are told the sample is \( p \)-type, and:
  - The electron density is \( n = 0 \).
  - The Hall Factor is \( r = 0 \).
- If we apply a voltage in the \( x \) direction of 10V, we measure a current of 500 \( \mu \)A.

If we apply a magnetic field of 0.5 T, we observe a Hall voltage of \( V_H = +30 \) mV.

- From this information, determine the hole mobility in this sample. Give your answer in cm\(^2\)/Vs.
- First calculate the resistivity of our sample. We would be given:
  \[
  \rho = \frac{Wd V_H}{L I}
  \]
- Enter values (stay in meters for now):
  \[
  \rho = \frac{1 \times 10^{-3} \times 1 \times 10^{-3}}{5 \times 10^{-3}} = \frac{10}{500 \times 10^{-6}} = 4 \, \Omega m
  \]
Example 3

• Now calculate Hall Factor \( (R_H) \). You would be given:

\[
R_H = \frac{V_H d}{IB_z}
\]

• Enter values:

\[
R_H = \frac{0.03 \times 10^{-3}}{500 \times 10^{-6} \times 0.5} = 0.12 \text{ m}^3\text{C}^{-1}
\]

• Finally, we can get the mobility from \( \rho \) and \( R_H \). This would also be given:

\[
\mu_H = \frac{|R_H|}{\rho}
\]

Example 3

• Enter values:

\[
\mu_H = \frac{0.12}{4}
\]

\[
\mu_H = 0.03 \text{ m}^2/\text{Vs}
\]

\[
\mu_H = 300 \text{ cm}^2/\text{Vs}
\]
Example 4

• Describe some of the problems with time-of-flight drift mobility measurements:
  • In addition to transport in an electric field, carriers also undergo diffusion, broadening the signal in space and time.
  • Injection barriers may significantly effect results.
  • Carrier velocity may not be linear with electric field strength over the distances studied.

Good Luck!