Lecture 17
Field-Effect Transistors 2
Schroder: Chapters 2, 4, 6

Announcements

Homework 4/6:
• Is online now.
• Due Monday May 21st at 10:00am.
• I will return it the following Monday (28th May).

Homework 5/6:
• It will be online later today.
• This one will be a little different, and will basically be a data analysis exercise.
• It will still carry the same weight (7.5% of overall grade).
• Due Monday May 28th at 10:00am.
• I will return it the following Monday (4th June).
Other Sources of Information

- It turns out that the information on FETs in Schroder is not well organized. It is spread across multiple chapters.
- There are some other good sources of information on this subject if you are interested:
- Review Articles:
  - https://pubs.acs.org/doi/abs/10.1021/cr0501543
  - https://pubs.acs.org/doi/abs/10.1021/cm049391x
- Books:
- Course
  - ECE599 – Thin Film Electronics, John Labram, Fall 2018.

Lecture 17
- The Gradual Channel Approximation.
- Evaluating Field-Effect Mobility.
The Gradual Channel Approximation

Modelling FETs

• There is only one main model for describing FETs: The Gradual Channel Approximation.
• It is used throughout semiconductor research and industry.
• It does not depend on device architecture, geometry or semiconductor material type (generally).
• It is used to extract parameters of interest from current-voltage characteristics.
• We are going to derive it from first principles.
• We will derive the model for an n-type FET, but as usual the theory is general.
Charge Injection

- Let’s take a look at our FET again.
- We ground our source electrode.
- This acts as our source of charge carriers.
- First let us consider what happens when we apply a positive gate voltage ($V_G$).

If the energy offsets are correct,

- Last time we saw that carriers will be injected.
Charge Injection

- Initially charges will accumulate under the injecting electrode (source).
- But these carriers will be repelled, making space for more carriers.
- Eventually uniform charge will accumulate across the interface.
- As a first approximation, the electrode overlap area doesn’t matter.

Capacitance

- We can identify this situation as a capacitor.
- If we know the capacitance of the dielectric, $C$ (we can measure or calculate this), then we can determine the charge accumulated ($Q'$) for a certain $V_G$:

$$Q' = CV_G$$

The next slide explains why we use a prime (’).
Capacitance / Area

- Because it makes more sense when comparing devices of different size, we instead will talk about charge per unit area \( (Q = Q' / A) \), and capacitance per unit area \( (C_{ox} = C / A) \):

\[
Q = C_{ox} V_G
\]

- \( Q \) has units of C/cm\(^2\).
- \( C_{ox} \) has units of F/cm\(^2\).

Trapped Charge

- Recall that for capacitors, a lot of charge is not mobile.
- Some injected charge will fill traps and be immobile.
- Some already-trapped charge will be released and become mobile.
- Also, the semiconductor, dielectric, and the interface may be not be electrically neutral at \( V_G \).
Threshold Voltage

- We are interested in the density of mobile charges at the interface.
  - I.e. those that are capable of transport and representative of device operation.
- We define the mobile charge per unit area as $Q_{mob}$.
- As with the flatband voltage in MOS capacitors, trapped charge will shift voltage characteristics.
- We can quantify this voltage offset by a threshold voltage: $V_T$
- And then say: $Q = C_{ox} V_G \Rightarrow Q_{mob} = C_{ox} (V_G - V_T)$

Applied Drain Voltage

$Q_{mob} = C_{ox} (V_G - V_T)$

- This equation is valid for an FET with no applied drain bias (since it is basically a capacitor).
- However when a bias is applied, there is a clearly a variation in charge density as a function of position.
**Applied Drain Voltage**

- We can account for the application of a drain voltage by adding an extra voltage term \( V(x) \) to our equation for mobile charge density:

\[
Q_{mob}(x) = C_{ox}(V_G - V_T - V(x))
\]

**Applied Drain Voltage**

- Now let's take a top-down look at our FET:
- Define our semiconducting channel to have a length \( L \) and a width \( W \) (sometimes called \( Z \))
- We assume that \( W \gg L \) and hence can neglect edge effects.
- At \( x = 0 \), \( V(0) = 0 \) (it is at the source terminal).
- At \( L = 0 \), \( V(L) = V_D \) (it is at the drain terminal).
Channel Conductivity

\[ Q_{mob}(x) = C_{ox}(V_G - V_T - V(x)) \]

- This tells us the charge density changes as a function of position in the channel \((x)\).
- Therefore the conductivity of the channel must also depend on position: \(\sigma = \sigma(x)\).
- For a sample with a homogenous distribution of carriers, we would evaluate conductivity as a function of charge density using the expression:

\[ \sigma = e(\mu_e n + \mu_h p) \]

- For an n-type sample this would simplify to

\[ \sigma = e\mu n \]

- Let’s take another look at our channel. For now, we say the thickness is \(D\):
Split up the Channel in $x$

- But we know the carrier density is not uniform.

- Instead of considering $L$, we can instead consider a very small slice of the channel, of length $\delta x$.
- We will make the approximation that in this slice the charge distribution is uniform.
- As $\delta x \to 0$, this becomes a valid assumption.

Channel Conductivity

- Consider carrier density in this strip of the channel:
  - Say this is now valid: $\sigma(x) = e\mu n(x)$
  - If $n$ is the carrier density, let $N$ be the number of carriers.

- Therefore in a volume $V$:
  $$n(x) = \frac{N(x)}{V} = \frac{N(x)}{WD\delta x}$$

- Put back into conductivity:
  $$\sigma(x) = \frac{e\mu N(x)}{WD\delta x}$$
Channel Conductivity

- We identify $eN(x)$ as total charge: $Q'_{mob}(x) = eN(x)$

$$\sigma(x) = \frac{\mu Q'_{mob}(x)}{WD\delta x}$$

- We also have an equation for 2-dimensional charge density as a function of position:

$$Q_{mob}(x) = C_{ox}(V_G - V_T - V(x))$$

- We can write 2-dimensional charge density ($Q_{mob}$) in terms of total charge ($Q'_{mob}$)

$$Q_{mob}(x) = \frac{Q'_{mob}(x)}{W\delta x} \quad \Rightarrow \quad Q'_{mob}(x) = Q_{mob}(x)W\delta x$$

- Substitute $Q'_{mob}(x)$ into our equation for $\sigma(x)$

$$\sigma(x) = \frac{\mu Q_{mob}(x)W\delta x}{WD\delta x}$$

$$\sigma(x) = \frac{\mu Q_{mob}(x)}{D}$$

- Use our equation for resistivity / conductance from Lecture 2:
Channel Resistance

- In our system we identify the cross-sectional area as: $WD$.
- Identify length as $\delta x$.

$$\sigma = \frac{1}{\rho} = \frac{\delta x}{RWD} \quad \Rightarrow \quad \delta R = \frac{\delta x}{\sigma(x)WD}$$

- Note: We use $\delta R$ because we are talking about a small part of the channel, rather than the resistance ($R$) of the whole channel.
- Substitute in $\sigma(x)$:

$$\delta R = \frac{D \delta x}{\mu Q_{mob}(x)WD} \quad \Rightarrow \quad \delta R = \frac{\delta x}{\mu Q_{mob}(x)W}$$

Channel Resistance

$$\delta R = \frac{\delta x}{\mu Q_{mob}(x)W}$$

- Now consider what happens to the change in resistance as $\delta x \to 0$:

$$dR = \frac{dx}{\mu Q_{mob}(x)W}$$

- We can use the differential form of Ohm’s law to describe the current arising from changes in resistance with respect changes in voltage:

$$I_D = \frac{dV}{dR}$$
Source Drain Current

- We call the current flowing in our device the source-drain current. Often this is shortened to drain current, labelled: \( I_D \).

\[
I_D = \frac{dV}{dR} \quad dR = \frac{dx}{\mu Q_{mob}(x)W}
\]

- Combine these two differential equations:

\[
I_D = \left( \frac{dV}{dx} \right) \mu Q_{mob}(x)W
\]

- Substitute in our equation for \( Q_{mob}(x) \):

\[
I_D = \frac{dV}{dx} \mu C_{ox}(V_G - V_T - V(x))W
\]

Source Drain Current

\[
I_D = \frac{dV}{dx} \mu C_{ox}(V_G - V_T - V(x))W
\]

- Separate the differentials:

\[
I_D dx = \mu C_{ox}(V_G - V_T - V(x))W dV
\]

- Which we can evaluate by integration:

\[
\int_{?}^{?} I_D dx = \int_{?}^{?} \mu C_{ox}(V_G - V_T - V(x))W dV
\]

- We just need to chose the limits.
Channel Integral

• Look at our whole channel again:

\[ V(0) = 0 \quad V(L) = V_D \]

• We need to integrate over the whole channel.
• So we need to integrate from \( x = 0 \) (where \( V(x) = 0 \)) to \( x = L \) (where \( V(x) = V_D \)).

Source Drain Current

• Put in these limits:

\[ \int_0^L I_D dx = \int_0^{V_D} \mu C_{ox} (V_G - V_T - V(x)) W dV \]

• Current does not vary with position, so:

\[ \int_0^L I_D dx = [I_D x]_0^L = I_D (L - 0) = I_D L \]

• Hence we can say:

\[ I_D = \frac{W}{L} \mu C_{ox} \int_0^{V_D} (V_G - V_T - V(x)) dV \]
Source Drain Current

\[ I_D = \frac{W}{L} \mu C_{ox} \int_0^{V_D} (V_G - V_T - V(x))dV \]

- \( V_G \) and \( V_T \) are just constants. So:

\[ I_D = \frac{W}{L} \mu C_{ox} \left[ V_G V - V_T V - \frac{V^2}{2} \right]_0^{V_D} \]

\[ I_D = \frac{W}{L} \mu C_{ox} \left( V_G V_D - V_T V_D - \frac{V_D^2}{2} \right) \]

\[ I_D = \frac{W}{L} \mu C_{ox} \left[ (V_G - V_T)V_D - \frac{V_D^2}{2} \right] \]

Evaluating Field-Effect Mobility
Source Drain Current

• The “master equation” of the gradual channel approximation is:

\[ I_D = \frac{W}{L} \mu C_{ox} \left[ (V_G - V_T)V_D - \frac{V_D^2}{2} \right] \]

• This can be used to approximate charge carrier mobility from current-voltage characteristics.

• In Lecture 10 we talked about why mobility it is important:
  • Switching speed.
  • Amplification.

Mobility in Research

• A large motivation for a lot of work in emerging electronics is to improve charge carrier mobility.

Labram et. al. Small 11, 3472 (2015)
Warning

• Recently there has been some controversy over reported mobilities in the literature.
• A number of groups are reporting inaccurate (too high) mobilities.
  • Either using gradual channel approximation (GCA) incorrectly.
  • Or applying it to situations when the assumptions we made are not valid.

https://www.nature.com/articles/ncomms10908
http://science.sciencemag.org/content/352/6293/1521
https://www.nature.com/articles/nmat5035

Output Characteristics

• There are two main ways in which mobility is extracted from $IV$ characteristics.
  • In the linear regime $\rightarrow$ linear mobility ($\mu_{\text{lin}}$).
  • In saturation regime $\rightarrow$ saturation mobility ($\mu_{\text{sat}}$).
• Recall what output curves look like:
Output Characteristics

- Recall, this is due to the channel being “pinched-off”.

Linear Regime

- Recall, for the device to have ~linear characteristics we require the field pointing ~down → very small $V_D$.
Linear Regime

• We consider the FET in the linear regime when:

\[ |V_D| \ll |V_G - V_T| \]

• We can use this to say:

\[ (V_G - V_T)V_D \gg \frac{V_D^2}{2} \]

• Apply this approximation to the GCA Equation:

\[ I_D = \frac{W}{L} \mu_{\text{lin}} C_{ox} (V_G - V_T)V_D \]

• \( \mu_{\text{lin}} \) denotes this is only valid in the linear regime.

Linear Regime

\[ I_D = \frac{W}{L} \mu_{\text{lin}} C_{ox} (V_G - V_T)V_D \]

• We actually tend to evaluate \( \mu \) from transfer characteristics, rather than output characteristics.

• This is because it turns out we don’t need to know \( V_T \).
Linear Regime

- The way this is usual done is by numerically differentiating the transfer curve.
- Recall that the transfer curve is $I_D$ plotted against $V_G$:

$$I_D = \frac{W}{L} \mu_{\text{lin}} C_{\text{ox}} (V_G - V_T) V_D$$

$$\frac{dI_D}{dV_G} = \frac{W}{L} \mu_{\text{lin}} C_{\text{ox}} V_D$$

- Linear Mobility

$$\mu_{\text{lin}} = \frac{L}{WC_{\text{ox}} V_D} \frac{dI_D}{dV_G}$$

- Notice, we don’t need to know $V_T$ now.
- We just need to know the channel dimensions (length and width), and the dielectric capacitance.
- These are normally very easy to determine.
- It is important to emphasize that this is only valid in the linear regime.
- Hence only if:

$$|V_D| \ll |V_G - V_T|$$
Numerical Differentiation

- You are all familiar with differentiating mathematical functions:
  \[ y(x) = x^2 \quad \Rightarrow \quad \frac{dy}{dx} = 2x \]
- But the concept of differentiating real data may be new to some of you.
- Numerical differentiation is important to FET mobility analysis.
  - And also many other techniques.
- If you have experimental data of \( I_D \) vs \( V_G \), how do you determine \( \frac{dI_D}{dV_G} \)?

When going from analytical functions to real data, we have to consider derivatives as difference equations:

\[ \frac{dy}{dx} \xrightarrow{\text{\Delta}} \frac{\Delta y}{\Delta x} \]

Where \( \Delta y \) and \( \Delta x \) are now finite steps.

To find the derivative at a data point \( i \) we need to consider adjacent points \( i - 1 \) and \( i + 1 \).
Linear Example Data

• Here is what some real data looks like differentiated:

![Graph showing linear example data](image)

Linear Mobility

• In the linear regime we can only consider data where \(|V_D| \ll |V_G - V_T|\).
• Normally some average is taken in the correct part of the curve.
• Here:
  \[
  \frac{dI_D}{dV_G} \approx 1 \times 10^{-6} \text{A/V}
  \]
• The rest of the parameters are known:
  • \(V_D, L, W, C_{ox}\).

\[
\mu_{\text{lin}} = \frac{L}{WC_{ox}V_D} \frac{dI_D}{dV_G}
\]
Saturation Regime

• Recall, if $V_D$ is very large we get saturation behavior.

After pinch-off (saturation), a depletion region forms by the drain electrode.

• Here there are very few carriers, and this region is very resistive.

• Pinch-off occurs when $V_D = V_G - V_T$. 
Saturation Regime

- As the source-drain voltage is increased further the pinch-off point moves towards the source electrode and the depletion region will increase in size.
- The voltage drop between the source and the pinch-off point will remain constant at $V(x) = V_G - V_T$.

\[ V_D = \text{Constant} \]

\[ V(x) = V_G - V_T \]

- The additional voltage applied is dropped across the high-resistance depletion zone.
- Since carriers are only present in the left part of the channel, this is the effective device.
- The device behaves as if $V_D = (V_G - V_T)$ = constant.
Saturation Regime

- We consider the FET in the saturation regime when:
  \[ |V_D| \gg |V_G - V_T| \]
- The device behaves as if the voltage is constant at:
  \[ V_D = V_G - V_T \]
- Apply this approximation to the GCA Equation:
  \[ I_D = \frac{W}{2L} \mu_{sat} C_{ox} (V_G - V_T)^2 \]
- \( \mu_{sat} \) denotes this is only valid in the saturation regime.

Notice we now have \( I_D \propto V_G^2 \).

\[ I_D = \frac{W}{2L} \mu_{sat} C_{ox} (V_G - V_T)^2 \]

So we can’t just differentiate the function as we did for the linear mobility.

It turns out we have 2 options to get mobility.

**Approach 1**
- Take square root of \( I_D \) and differentiate that.

**Approach 2**
- Differentiate \( I_D \) twice.
Approach 1

\[ I_D = \frac{W}{2L} \mu_{\text{sat}} C_{ox} (V_G - V_T)^2 \]

- Take square root:

\[ \sqrt{I_D} = (V_G - V_T) \sqrt{\frac{W}{2L} \mu_{\text{sat}} C_{ox}} \]

- Now take 1st derivative:

\[ \frac{d}{dV_G} \sqrt{I_D} = \sqrt{\frac{W}{2L} \mu_{\text{sat}} C_{ox}} \]

Approach 1

\[ \frac{d}{dV_G} \sqrt{I_D} = \sqrt{\frac{W}{2L} \mu_{\text{sat}} C_{ox}} \]

- Square both sides:

\[ \left( \frac{d}{dV_G} \sqrt{I_D} \right)^2 = \frac{W}{2L} \mu_{\text{sat}} C_{ox} \]

\[ \mu_{\text{sat}} = \frac{2L}{W C_{ox}} \left( \frac{d}{dV_G} \sqrt{I_D} \right)^2 \]
Approach 2

\[ I_D = \frac{W}{2L} \mu_{\text{sat}} C_{\text{ox}} (V_G - V_T)^2 \]

- Multiply out:

\[ I_D = \frac{W}{2L} \mu_{\text{sat}} C_{\text{ox}} (V_G - V_T)(V_G - V_T) \]

\[ I_D = \frac{W}{2L} \mu_{\text{sat}} C_{\text{ox}} (V_G^2 - 2V_G V_T + V_T^2) \]

- Take 1\text{st} derivative:

\[ \frac{dI_D}{dV_G} = \frac{W}{2L} \mu_{\text{sat}} C_{\text{ox}} (2V_G - 2V_T) \]

Approach 2

\[ \frac{dI_D}{dV_G} = \frac{W}{2L} \mu_{\text{sat}} C_{\text{ox}} (2V_G - 2V_T) \]

- Take 2\text{nd} derivative:

\[ \frac{d^2 I_D}{dV_G^2} = \frac{W}{2L} \mu_{\text{sat}} C_{\text{ox}} (2) \]

\[ \frac{d^2 I_D}{dV_G^2} = \frac{W}{L} \mu_{\text{sat}} C_{\text{ox}} \]

- Re-arrange:

\[ \mu_{\text{sat}} = \frac{L}{W C_{\text{ox}} dV_G^2} \frac{d^2 I_D}{dV_G^2} \]
Saturation Example Data

- Here is what some real data looks like differentiated:

\[ (\frac{d\sqrt{I_D}}{dV_G})^2 \]

\[ d^2I_D \frac{dV_G}{dV_G}^2 \]

\[ V_D = 100V \]

Saturation Mobility

- Here we can only consider data where \(|V_D| \gg |V_G - V_T|\).
- Extracting mobility from saturation can be a little bit more subjective than linear mobility.
- Normally we look at the curve where the device is on \((V_G \gg V_T)\), but also in saturation.
Next Time

- Final Lecture on FETs
- Series Resistance

- Threshold voltage $I_{DS}$
- A few other parameters