Lecture 8
Carrier Lifetime
Schroder: Chapter 7

Announcements

Homework 2/6:

- Is online now.
- Due Monday 30th April at the start of the lecture (10:00am).
- I will return it one week later (7th May).
- I will put the solutions online when I receive all the homeworks, so you can use it for exam preparation.
Announcements

Midterm Exam:
• Friday May 4th at 10:00am in STAG113
• Exam will last 45 minutes.
  • The exam will start exactly at 10:00am!
• Closed book and closed notes.
  • You can, and are expected to, use a calculator.
• Choose 2 out of 3 questions.
  • If you answer 3 I will take best 2 scores.
• It will contribute 25% of overall grade for class.
• The exam will material covered in lectures 2-10 (inclusive).

Announcements

Midterm Exam:
• There will be a Review Lecture on Wednesday May 2nd.
  • I will go through examples.
  • I will make it clear what you should know.
  • All constants will be provided:

<table>
<thead>
<tr>
<th>Useful Constants:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Name</strong></td>
</tr>
<tr>
<td>Conversion from Celcius to Kelvin</td>
</tr>
<tr>
<td>Boltzmann Constant</td>
</tr>
<tr>
<td>Atomic mass units</td>
</tr>
<tr>
<td>Mass of electron in kg</td>
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<tr>
<td>Joules per Electron Volt</td>
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<tr>
<td>Plate Constant</td>
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<tr>
<td>Fundamental mass of charge</td>
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<tr>
<td>Speed of light in vacuum</td>
</tr>
<tr>
<td>Lattice constant of GaAs</td>
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<tr>
<td>Diffusion prefactor for phosphorus diffusion in silicon</td>
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<tr>
<td>Activation energy for phosphorus diffusion in silicon</td>
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<tr>
<td>Mass of hole in m</td>
</tr>
<tr>
<td>Mass of phosphorus atom</td>
</tr>
<tr>
<td>Mass of oxygen atom</td>
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<tr>
<td>Mass of hydrogen atom</td>
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<tr>
<td>Mass of silicon atom</td>
</tr>
</tbody>
</table>
Announcements

Midterm Exam Style:

I will put online early (Monday April 30th).
So you can use it as midterm preparation if you want.
Due Monday 14th May at the start of the lecture (10:00am).
Will cover lectures 8-10.
Lecture 8

• Lifetime Overview.
• Shockley-Read-Hall Recombination.
• Radiative Recombination.
• Auger Recombination.
• Lifetime Revisited.

Lifetime Overview
**Lifetime**

- In electronic devices we inject carriers and typically expect them to travel as far as possible.

- In an idealized situation we expect every carrier to reach the opposite electrode and be extracted.
- Today’s lecture is concerned with the fact that this is not the case.

- Carriers that are being transported can **recombine** while in a device.

- The average time that carriers last is referred to as the **lifetime**.
Lifetime

- Values of recombination and lifetime heavily dependent on the situation.
  - They are probably one of the most ambiguous parameters in the field.
- Lifetime depends on:
  - Material type.
  - Carrier density.
  - Temperature.
  - Illumination density.
  - Point defect density.
  - Device Type.
  - Surface properties.
  - Grain boundaries.
  - Bias conditions.
  - Many others...

Recombination Lifetime

- In this lecture we provide a simplified overview.
- According to Schroder there are two types of lifetime:

\[ \tau_r \quad \text{Recombination Lifetime} \]

\[ \tau_g \quad \text{Generation Lifetime} \]

- Generation lifetime is basically the time it takes to thermally create carriers. We will not study it today.
Recombination Lifetime

• Today we are talking about electronic devices here (mainly transistors).
  \[ V = \pm \]
  Gate
  \[ h^+ \quad h^+ \quad h^+ \]
  Source
  Drain
  p-type

• In this context we are talking about the **lifetime of minority carriers in a sea of majority carriers**.

• For example, we are concerned with the **lifetime of electrons in a p-type semiconductor**.

Aside: Solar Cells

• Terminology depends on device type.

• In solar cells carrier lifetime is **very important**.

• Both holes and electrons need enough time to make it to electrodes.

• Lifetime in the context of solar cells is normally specified by carrier type: \( \tau_e \) or \( \tau_h \).

• If the term “lifetime” is used without specifying a carrier type it will refer to the carrier with the longest lifetime.
Lifetime

- But today we are talking about the recombination lifetime ($\tau_r$) of minority carriers in electronic devices.
- We will typically talk about electrons in a p-type semiconductor, but the concepts are identical to the opposite situation.
- The recombination lifetime is separated into three terms:

$$\frac{1}{\tau_r} = \frac{1}{\tau_{SRH}} + \frac{1}{\tau_{rad}} + \frac{1}{\tau_{Auger}}$$

Lifetime

- A convenient way to visualize these three processes is to consider the energy band diagram.
- Consider how energy is conserved in each case.
Lifetime

• We can also consider the number of charge-carriers involved in the process.

SRH (monomolecular)  Radiative (bimolecular)  Auger (trimolecular)

Shockley-Read-Hall Recombination
SRH Recombination

- There are four types of trap emission and capture processes. Same notation as Lecture 6.

\[ E_T \quad e^- \quad c_n \quad e_n \quad (b) \quad e^- \quad E_C \quad h^+ \quad h^+ \quad (c) \quad (d) \quad e^- \quad h^+ \quad c_p \quad e_p \]

- Capture coefficient for electrons
- Emission coefficient for electrons
- Capture coefficient for holes
- Emission coefficient for holes

Net Capture Rate

- The net rate of electron capture into a trap, \( U_n \), is given by the rate of electron trap capture minus the rate of electron trap emission:

\[ U_n = r_a - r_b \]

- Recall from Lecture 6 we can describe the electron capture rate by:

\[ r_a = \sigma n v_{th} n N_T (1 - f_T) \]

- And the electron emission rate by:

\[ r_b = e_n N_T f_T \]
Parameters

\[ r_a = \sigma_n \nu_{th}^e n N_T (1 - f_T) \quad r_b = e_n N_T f_T \]

- \( r_a \) = Rate of electron capture.
- \( \sigma_n \) = Electron capture cross-section (e.g. cm\(^2\)).
- \( \nu_{th}^e \) = Electron thermal velocity.
- \( n \) = Delocalized electron density in conduction band.
- \( N_T (1 - f_T) \) = Number density of empty traps.
- \( r_b \) = Rate of electron emission.
- \( e_n \) = Emission coefficient for electrons
- \( N_T f_T \) = Number density of filled traps.

e\(^-\) Emission coefficient

- We also know from Lecture 6, that we can describe the emission coefficient in terms of temperature and band energies:

\[ e_n = \sigma_n \nu_{th}^e N_c \exp \left( -\frac{E_C - E_T}{k_B T} \right) \]

- \( N_c \) = Number density conduction band states.
- \( E_C \) = Conduction band energy.
- \( E_T \) = Trap state energy.
- \( k_B \) = Boltzmann Constant.
- \( T \) = Temperature.
Net Capture Rate

- Substituting this back into our equation for net capture rate:

\[
r_a = \sigma_n v_{th} n N_T (1 - f_T)
\]

\[
r_b = e_n N_T f_T
\]

\[
U_n = r_a - r_b = \sigma_n v_{th} n N_T (1 - f_T) - e_n N_T f_T
\]

\[
e_n = \sigma_n v_{th} N_c \exp\left(-\frac{E_C - E_T}{k_b T}\right)
\]

\[
U_n = \sigma_n v_{th} \left[n N_T (1 - f_T) - N_c \exp\left(-\frac{E_C - E_T}{k_b T}\right) N_T f_T\right]
\]

Net Capture Rate

\[
U_n = \sigma_n v_{th} \left[n N_T (1 - f_T) - N_c \exp\left(-\frac{E_C - E_T}{k_b T}\right) N_T f_T\right]
\]

- We can define the following variable to simplify the expression for \(U_n\):

\[
n_1 = N_c \exp\left(-\frac{E_C - E_T}{k_b T}\right)
\]

- Recognize that \(n_1\) corresponds to the concentration of electrons in the conduction band when the Fermi level is located at the trap energy level. Or, generally:

\[
n_1 = n_i \exp\left(-\frac{E_i - E_T}{k_b T}\right)
\]

\[
n_1^2 = p_1 n_1
\]
Net Capture Rate - Holes

- We can (but won’t) derive a similar expression for the net capture rate of holes:

\[ U_p = \sigma_p v_{th}^h N_T [p f_T - p_1 (1 - f_T)] \]

- \( \sigma_p \) = Hole capture cross-section (e.g. cm\(^2\)).
- \( v_{th}^h \) = Hole thermal velocity.
- \( p \) = Delocalized hole density in valence band.
- \( p_1 = \frac{N_V}{n_i} \exp \left( -\frac{E_T - E_i}{k_b T} \right) \)
- \( p_1 \) = concentration of holes in the valence band when the Fermi level is positioned at the trap energy level.

Steady-State Trap Occupancy

- Consider the steady-state situation in which the net rates of electron and hole capture are equal:

\[ U_n = U_p \]

- Under these conditions we can define the steady-state trap occupancy: \( f_T \rightarrow f_T^{(ss)} \).
- This turns out to be:

\[ f_T^{(ss)} = \frac{\sigma_n n + \sigma_p p_1}{\sigma_n (n + n_1) + \sigma_p (p + p_1)} \]
- Where we have equated hole and electron thermal velocities:

\[ v_{th}^e = v_{th}^h = v_{th} \]
SRH Capture Rate

- We define the Shockley-Read-Hall capture rate ($U_{SRH}$) as the **steady-state** situation in which the net rates of electron and hole capture are equal:

  \[ U_n = U_p = U_{SRH} \]

- It turns out this can be expressed as:

  \[ U_{SRH} = \frac{\sigma_n \sigma_p v_{th} N_T (pn - n_i^2)}{\sigma_n (n + n_1) + \sigma_p (p + p_1)} \]

- This is a very important relationship which is used extensively in processes involving SRH generation and recombination.

Excess Carrier Concentration

- If we go back to our steady-state trap occupancy factor:

  \[ f_T^{(ss)} = \frac{\sigma_n n + \sigma_p p_1}{\sigma_n (n + n_1) + \sigma_p (p + p_1)} \]

- We identify that this is not equal to the Fermi-Dirac distribution:

  \[ f_T^{(ss)} \neq \frac{1}{1 + \exp \left( \frac{E_T - E_F}{k_b T} \right)} \]

- $f_T^{(ss)}$ is a **non-equilibrium** occupancy.

- Because carriers are constantly being trapped and detrapped they are not Fermi-distributed.
Excess Carrier Concentration

- We can describe our non-equilibrium carrier distributions as perturbations from equilibrium (i.e. without the presence of the trap):
  
  \[ n = n_0 + \Delta n \quad \text{and} \quad p = p_0 + \Delta p \]

- \( n \) = Non-equilibrium electron density.
- \( p \) = Non-equilibrium hole density.
- \( n_0 \) = Equilibrium electron density.
- \( p_0 \) = Equilibrium hole density.
- \( \Delta n \) = Excess electron density.
- \( \Delta p \) = Excess hole density.

This formulation allows us to re-write \( U_{SRH} \).

\[
U_{SRH} = \frac{\sigma_n \sigma_p v_{th} N_T (pn - n_i^2)}{\sigma_n (n + n_1) + \sigma_p (p + p_1)}
\]

- Start by enforcing charge neutrality on our excess carriers:
  \[
  \Delta n = \Delta p
  \]

- Turns out this is a valid assumption as long as one of: \( n_0, p_0, n_1, p_1 \) is large compared to \( N_T \).
  - See the Schroder or W. Shockley and W.E. Read, Phys. Rev. 87, 835 (1952) for why this is the case.
**Excess Carrier Concentration**

- Look at $pn - n_i^2$ term of $U_{SRH}$:

  $$pn - n_i^2 = (n_0 + \Delta n)(p_0 + \Delta p) - n_i^2$$

  $$pn - n_i^2 = n_0p_0 + n_0\Delta p + n_0\Delta p + n_0\Delta n - n_i^2$$

- Identify the equilibrium carrier density as the intrinsic carrier density:

  $$n_0p_0 = n_i^2$$

  $$pn - n_i^2 = n_0\Delta p + n_0\Delta p_0 + n_0\Delta n$$

- Enforce $\Delta n = \Delta p$:

  $$pn - n_i^2 = n_0\Delta n + n_0\Delta p_0 + n_0\Delta n$$

  $$pn - n_i^2 = (n_0 + p_0 + \Delta n)\Delta n$$

**SRH lifetime**

- Substitute $pn - n_i^2$ back into $U_{SRH}$:

  $$U_{SRH} = \frac{\sigma_n\sigma_p v_{th} N_T (pn - n_i^2)}{\sigma_n (n + n_1) + \sigma_p (p + p_1)}$$

  $$U_{SRH} = \frac{\sigma_n\sigma_p v_{th} N_T \Delta n (n_0 + p_0 + \Delta n)}{\sigma_n (n + n_1) + \sigma_p (p + p_1)}$$

- And define the **SRH lifetime**:

  $$\tau_{SRH} = \frac{\tau_p(n_0 + n_1 + \Delta n) + \tau_n(p_0 + p_1 + \Delta p)}{p_0 + n_0 + \Delta n}$$

  $$\tau_p = \frac{1}{\sigma_p v_{th} N_T}$$

  $$\tau_n = \frac{1}{\sigma_n v_{th} N_T}$$

  $$U_{SRH} = \frac{\Delta n}{\tau_{SRH}}$$
Important Limits

- **Low-Level Injection:**
  - The excess minority carrier concentration is much less than the equilibrium (intrinsic) majority carrier concentration.
  - For p-type material:
    $$ \Delta n \ll p_0 $$
    $$ \tau_{SRH}^{(ll)} \approx \tau_n $$
  - For n-type material:
    $$ \Delta p \ll n_0 $$
    $$ \tau_{SRH}^{(ll)} \approx \tau_p $$

- **Very High Level Injection:**
  - Although this approximation is extensively employed for low-level injection conditions, note that it is only valid when the material is strongly of one type (e.g., when $p_0 \gg n_0$ for a p-type semiconductor) and when the trap location is sufficiently deep in the bandgap (e.g., when $p_0 \gg p_1$ and $p_0 \gg n_1$).
  - For p-type material:
    $$ \Delta n \gg p_0 $$
    $$ \tau_{SRH}^{(hl)} \approx \tau_n + \tau_p $$
  - For n-type material:
    $$ \Delta p \gg n_0 $$
    $$ \tau_{SRH}^{(hl)} \approx \tau_n + \tau_p $$
Radiative Recombination

Overview

- Sometimes called bimolecular recombination.
- Basically just a hole and electron annihilating:

Energy Picture

Spatial Picture
Radiative Recombination

- This is an important process for a direct bandgap semiconductor, but is usually of no importance in an indirect semiconductor, unless it is extraordinarily pure.

- In an indirect semiconductor, a transfer of momentum is required for recombination (and generation).

Radiative Recombination

- The idea of radiative recombination is that an electron in the conduction band annihilates a hole in the valence band by a conduction-to-valence band recombination event.

- However, the potential energy of the electron and the hole has to be conserved.

- This is accomplished by the emission of a photon at a frequency corresponding to the energy of the bandgap

\[ \nu = \frac{E_G}{h} \]

Where:
- \( \nu \) is the frequency of the photon
- \( E_G \) is the energy of the bandgap
- \( h \) is Planck's constant

\[ \text{Band Gap} \]

\[ \text{"nu", } \nu \neq \nu \]

\[ \text{frequency} \]
Recombination Rate

• The rate of radiative recombination rate \( R_{rad} \) is proportional to a product of the non-equilibrium carrier densities

\[
R_{rad} = \alpha n p
\]

• It has units \( \text{cm}^{-3}\text{s}^{-1} \) (c.f. \( \text{s}^{-1} \) for SRH rate).

• We can show (but won’t) that:

\[
\alpha = \frac{R}{n_i^2}
\]

• \( R \) is the recombination rate in thermal equilibrium.

\[
R = \frac{32\pi^2 c^3 (k_B T)^4}{h^4} \int_0^\infty \frac{k \hat{n}^3 x^3}{e^x - 1} \, dx
\]

• \( R \) is the recombination rate in thermal equilibrium.

• \( c \) is the speed of light in a vacuum.

• \( k_B \) is the Boltzmann Constant.

• \( h \) is the Planck Constant.

• \( k \) is the imaginary part of the index of refraction (i.e., the extinction coefficient).

• \( \hat{n} \) is the index of refraction (i.e., the real part of the complex index of refraction).

Note: this normally is just labeled \( n \). I have put a hat on it here, to distinguish it from \( e^- \) density.
Recombination Rate

\[ R = \frac{32\pi^2 c^3 (k_B T)^4}{h^4} \int_0^\infty \frac{k\tilde{n}^3 x^3}{e^x - 1} dx \]

- Where:
  \[ x = \frac{h\nu}{k_B T} \]
  - Optical properties

- \( \nu \): Photon frequency.
- So \( x \) is just optical energy normalized for thermal energy.
- \( R \) depends on the optical properties of the material (i.e., \( \tilde{n} \) and \( k \)) and on the temperature

Replacing \( \alpha \) with \( R \) in the previous expression for the radiative recombination rate gives:

\[ R_{rad} = \alpha n p \]

\[ \alpha = \frac{R}{n_i^2} \]

- Replacing nonequilibrium carrier concentrations by their equivalent excess carrier concentration expressions can get this into a more useful form:

\[ n = n_0 + \Delta n \]
\[ p = p_0 + \Delta p \]
Recombination Rate

\[ n = n_0 + \Delta n \quad p = p_0 + \Delta p \]

\[ R_{rad} = \frac{R}{n_i^2} np \]

\[ R_{rad} = \frac{R}{n_i^2} (n_0 + \Delta n)(p_0 + \Delta p) \]

\[ R_{rad} = \frac{R}{n_i^2} (n_0p_0 + n_0\Delta p + p_0\Delta n + \Delta n\Delta p) \]

- Again, use \( n_i^2 = n_0p_0 \):

\[ R_{rad} = R + \frac{R}{n_i^2} (n_0\Delta p + p_0\Delta n + \Delta n\Delta p) \]

Net Rate of Recombination

- The net rate of radiative recombination is defined as:

\[ U_{rad} = R_{rad} - R = \frac{R}{n_i^2} (n_0\Delta p + p_0\Delta n + \Delta n\Delta p) \]

- This is the most general solution of radiative recombination.
- To make further progress we enforce charge neutrality: \( \Delta n = \Delta p \).

\[ U_{rad} = \frac{\Delta p}{\tau_{rad}} \]

- Where:

\[ \tau_{rad} = \frac{1}{B(p_0 + n_0 + \Delta n)} \]

\[ B = \frac{R}{n_i^2} \]
Radiative Recombination Coefficient

- The parameter $B$ is called the radiative recombination coefficient, and is typically quoted in units of cm$^3$s$^{-1}$.
- $B$ depends on the optical properties of the semiconductor of interest and the semiconductor.
- Typical values (at room temperature):

<table>
<thead>
<tr>
<th>Material</th>
<th>$B$ (cm$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>$4.73 \times 10^{-15}$</td>
</tr>
<tr>
<td>Ge</td>
<td>$5.2 \times 10^{-14}$</td>
</tr>
<tr>
<td>GaAs</td>
<td>$1.7 \times 10^{-10}$</td>
</tr>
<tr>
<td>GaP</td>
<td>$5.4 \times 10^{-14}$</td>
</tr>
<tr>
<td>InP</td>
<td>$1.8 \times 10^{-11}$</td>
</tr>
<tr>
<td>InSb</td>
<td>$4.6 \times 10^{-11}$</td>
</tr>
<tr>
<td>InGaAsP</td>
<td>$4 \times 10^{-10}$</td>
</tr>
</tbody>
</table>

Lower numbers = faster recombination

Higher numbers = slower recombination

High and Low-Level Limits

- Low-Level Injection:

$$\Delta n \ll p_0 \quad \Rightarrow \quad \tau_{rad}^{(ll)} \approx \frac{1}{Bp_0}$$

- Very High Level Injection:

$$\Delta n \gg p_0 \quad \Rightarrow \quad \tau_{rad}^{(hl)} \approx \frac{1}{B\Delta n}$$
Photon Recycling

- Before we conclude it is worth considering what happens to the photon that is emitted.
- In some systems (e.g. GaAs and perovskites) the photon can be re-absorbed: Photon Recycling.

Auger Recombination
Auger Recombination

- In a band-to-band Auger recombination process, conservation of potential energy associated with the annihilation of an electron-hole pair is achieved by transferring this potential energy to a third carrier, making it more energetic.

- There are two types of band-to-band Auger process, which can be distinguished as $eeh$ or $ehh$. 

![Diagram of Auger Recombination]

$$E_C$$
$$E_V$$

$eeh$

$ehh$
**eeh-Auger Recombination**

- First, we consider the *eeh*-Auger recombination process.
- Since we have two conduction band electrons and one valence band hole participate in such a process, we expect the rate \( R_{eeh} \) of such an Auger process to be proportional to the corresponding nonequilibrium carrier densities:

\[
R_{eeh} = C_n p n^2
\]

- Where \( C_n \) is the proportionality constant for *eeh*-Auger recombination.

**eeh-Auger Recombination**

- In equilibrium, the rate of *eeh*-Auger recombination and generation must be equal.
- Equivalently, the net rate of *eeh*-Auger recombination must be equal to zero in equilibrium.
- Recognizing this allows us to write the net rate of *eeh*-Auger recombination as:

\[
U_{eeh} = C_n (p n^2 - p_0 n_0^2)
\]
**eeh-Auger Recombination**

\[ U_{eeh} = C_n (pn^2 - p_0 n_0^2) \]

- Substituting nonequilibrium carrier concentrations by their equivalent excess carrier expressions and assuming charge neutrality:
  \[ n = n_0 + \Delta n \]
  \[ p = p_0 + \Delta p \]
  \[ \Delta n = \Delta p \]

- Leads to:
  \[ U_{eeh} = C_n (2p_0 n_0 + p_0 \Delta n + n_0^2 + 2n_0 \Delta n + (\Delta n^2) \Delta n) \]

**ehh-Auger Recombination**

- We can show, via a similar analysis that the net rate for electron-hole-hole Auger recombination:
  \[ U_{ehh} = C_p (2p_0 n_0 + n_0 \Delta n + p_0^2 + 2p_0 \Delta n + (\Delta n^2) \Delta n) \]

- Where \( C_p \) is the proportionality constant for ehh-Auger recombination.
Auger Lifetime

Finally, the Auger lifetime can be given as:

\[ \tau_{Auger} = \frac{\Delta n}{U_{eeh} + U_{ehh}} \]

- **Low-Level Injection:**
  \[ \Delta n \ll p_0 \]
  \[ \tau_{Auger}^{(ll)} \approx \frac{1}{C_p p_0^2} \]

- **Very High Level Injection:**
  \[ \Delta n \gg p_0 \]
  \[ \tau_{Auger}^{(hi)} \approx \frac{1}{(C_n + C_p)\Delta n^2} \]

Lifetime Revisited
Lifetime

- Recall that we began this lecture with the recombination lifetime, $\tau_r$, which is the average time that a nonequilibrium minority carrier exists in a sea of majority carriers before it recombines.

$$\frac{1}{\tau_r} = \frac{1}{\tau_{SRH}} + \frac{1}{\tau_{rad}} + \frac{1}{\tau_{Auger}}$$

- Most of this lecture has been devoted to obtaining expressions for: $\tau_{SRH}$, $\tau_{rad}$, $\tau_{Auger}$.
- Now that we have these expressions, we are able to assess the lifetime, usually the minority carrier lifetime, using these expressions.

Lifetime

- As a last thought, recognize that there are two types of semiconductor device types, **bipolar** and **unipolar**.
- Bipolar devices include pn junctions, bipolar junction transistors and thyristors.
- Minority carrier lifetime is extremely important for bipolar devices.
- In fact, bipolar devices may be categorized as devices whose basic operation is determined exclusively by minority carrier action.
- Unipolar devices include Schottky barriers and FETs.
Lifetime

- Minority carrier lifetime is not relevant for unipolar devices.
- However, majority carrier lifetime may be important for certain classes of unipolar devices, in which the semiconductor is lightly doped.

Next Time

- Surface Recombination.
- Photoconductance Decay Measurements