CS331 (Spring 2019): Introduction to Artificial Intelligence
Written Assignment #2

Date handed out: April 24, 2019
Date due: May 1, 2019 at 10:00
Total: 31 points

The written portion of this assignment is to be done individually. Please hand in a pdf on Canvas. Assignments done on a word processor are preferred but not mandatory. For hand written assignments, if we cannot read your writing, we cannot mark your assignment.

1. For each of the following statements about entailment, prove if it is true or false.
   a) \((A \land \neg A) \models True\) [2 points]
   True because \((A \land \neg A) \equiv False\) has no models and hence entails every sentence AND because \(True\) is true in all models and hence is entailed by every sentence.

   b) \((A \lor \neg A) \models False\) [2 points]
   False. \(False\) is not true in any models.

   c) \((A \iff B) \models (A \lor B)\) [3 points]
   False because the left-hand side has two models where it is true and the right-hand side is only true in one of those.

   d) \((A \land B) \models A \implies B\) [3 points]
   True because the one model in which the left-hand side is true also has the right-hand side being true.

2. Decide whether each of the following sentences is valid, unsatisfiable or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11.
   a) \(\neg Study \Rightarrow \neg Pass\) [2 points]
      \(\neg Study \lor \neg Pass\) is true for \(Study = True\) and false for \(Study = False\) and \(Pass = True\).
      Neither.

   b) \(((Study \land Rest) \Rightarrow Pass) \lor (((\neg Study \lor \neg Rest) \Rightarrow Pass)\) [4 points]
      \(\equiv \neg((S \land R) \lor P) \lor \neg((\neg S \lor \neg R) \lor P)\)
      \(\equiv \neg(S \land R) \lor P \lor (S \land R)\)
      Valid
3. Convert the following sentences to CNF. [5 points]

\[ S1. P \Leftrightarrow Q \]
\[ S2. (Q \land R) \Rightarrow S \]
\[ S3. (R \land S) \lor T \]

\[ S1a. \neg P \lor Q \]
\[ S1b. \neg Q \lor P \]
\[ S2. \neg Q \lor \neg R \lor S \]
\[ S3a. R \lor T \]
\[ S3b. S \lor T \]

4. Consider the following KB:

\[ S1. A \lor B \]
\[ S2. \neg B \lor \neg C \]
\[ S3. \neg C \lor D \]
\[ S4. B \lor E \]
\[ S5. D \lor F \]
\[ S6. E \lor \neg F \]
\[ S7. \neg D \]

a) Use the resolution algorithm to determine whether the following KB entails B. [5 points]

\[ S8. \neg B \]
\[ S9. A \quad (S7 + S1) \]
\[ S10. E \quad (S4 + S7) \]
\[ S11. A \lor \neg C \quad (S1 + S2) \]
\[ S12. \neg C \lor E \quad (S2 + S4) \]
\[ S13. D \lor E \quad (S5 + S6) \]
\[ S14. E \quad (S13 + S7) \]
\[ S15. \neg C \quad (S3 + S7) \]

No new clauses can be generated, so the KB does not entail B.

b) Use the resolution algorithm to determine whether the following KB entails \( \neg C \). [5 points]

\[ S7. C \]
\[ S8. D \quad (S7 + S3) \]
\[ S9. False \quad (empty clause) \quad (S8 + S7) \]

Since we found the empty clause, the KB entails \( \neg C \).