Question 1 [11 marks]:
Consider a pn-junction as shown in Figure 1(a).

\[ E = -\frac{d\phi}{dx} = \begin{cases} \left(-\frac{N_A e}{\varepsilon_0 \varepsilon_r}ight)(x + w_p) & -w_p < x < 0 \\ \left(-\frac{N_D e}{\varepsilon_0 \varepsilon_r}ight)(x - w_n) & 0 < x < w_n \\ 0 & \text{elsewhere} \end{cases} \]  

Where:
- \( E \) is electric field strength.
- \( \phi \) is electrostatic potential.
- \( x \) is position.
- \( N_A \) is acceptor density.
- \( N_D \) is the donor density.
- \( w_p \) is the width of the depletion region in the p-type semiconductor.
- \( w_n \) is the width of the depletion region in the n-type semiconductor.
- \( \varepsilon_0 \) is the vacuum permittivity.
- \( \varepsilon_r \) is the relative permittivity of the semiconductor.

If we define the electrostatic potential outside the p-type depletion region as zero, i.e. \( \phi(x = -w_p) = 0 \), then the electrostatic potential can be given by Equation 2.
\[ \phi = \begin{cases} 
\frac{N_A e}{2 \varepsilon_0 \varepsilon_r} (x + w_p)^2 & \text{if } -w_p < x < 0 \\
\Delta \phi_0 - \frac{e N_D}{2 \varepsilon_0 \varepsilon_r} (x - w_n)^2 & \text{if } 0 < x < w_n 
\end{cases} \] (2)

Where \( \Delta \phi_0 \) is given by Equation 3:

\[ \Delta \phi_0 = \frac{e}{2 \varepsilon_0 \varepsilon_r} \left( N_A w_p^2 + N_D w_n^2 \right) \] (3)

The depletion widths are given by Equations (4) and (5):

\[ w_p = \left( \frac{2 N_D \Delta \phi_0 \varepsilon_0 \varepsilon_r}{e N_A (N_D + N_A)} \right)^{1/2} \] (4)

\[ w_n = \left( \frac{2 N_A \varepsilon_0 \varepsilon_r \Delta \phi_0}{e N_D (N_D + N_A)} \right)^{1/2} \] (5)

a) Integrate Equation 1 to show that the electrostatic potential (\( \phi \)) as a function of position (\( x \)) is given by Equation 2. Hint: since we define the electrostatic potential as \( \phi = 0 \) at \( x = -w_p \), you should integrate the electric field from left-to-right across the whole junction, to the position you are interested in. [7 marks]

b) By asserting that electric field is continuous at \( x = 0 \), use Equations 1, 2 and 3 to show that the width of the depletion regions (\( w_p \) and \( w_n \)) are given by Equations 4 and 5. [4 marks]

Question 2 [6 marks]:

a) Figure 2 shows current-density-voltage characteristics (\( J \) vs \( V \)) of an example solar cell under illumination from \( P = 100 \text{ mW/cm}^2 \) light. This \( J \) vs \( V \) data is available to download using the link here. Using this data, determine the power conversion efficiency of this solar cell. Give your answer in percent. [6 marks]

![Figure 2](image-url)
Question 3 [8 marks]:

a) When we use the word “oxide” in the context of metal-oxide semiconductor (MOS) capacitors, what type of electronic properties do we mean? [1 mark]

b) If we operate an n-type MOS capacitor in accumulation mode, what type of charge carriers would you expect to observe at the interface between the semiconductor and insulator? [1 mark]

c) If we operate an n-type MOS capacitor in depletion mode, what type of charge carriers would you expect to observe at the interface between the semiconductor and insulator? [1 mark]

d) If we operate an n-type MOS capacitor in inversion mode, what type of charge carriers would you expect to observe at the interface between the semiconductor and insulator? [1 mark]

e) Figure 3 shows the capacitance per unit area of a MOS capacitor measured as a function of applied voltage (V) using a low frequency probe and a high frequency probe. From this data determine the capacitance the depletion region of this capacitor. Give your answer in μF/cm². [2 marks]

f) Say we are carrying out a capacitance measurement on a silicon-based MOS capacitor. For this capacitor, the thermal generation time is known to be 1 µs, the depletion width can be approximated to be 500 nm, and the capacitance per unit area of the oxide is 20 nF/cm². The intrinsic carrier concentration of silicon is $10^{10}$ cm⁻³. Determine the voltage ramp-rate required to observe inversion in this semiconductor. Give your answer in V/s. [2 marks]