Lecture 24
Review for Final Exam

Lecture 12
Diodes
Example 1

• Sketch the current-voltage characteristics of a diode, indicating clearly:
  a). The forward voltage.
  b). The breakdown voltage.
  c). Reverse saturation current.
• Plot on linear-linear axes.

Example 1
• This something you will have to know:

![Diagram showing diode characteristics with labels for forward voltage \( V_F \), breakdown voltage \( V_{BR} \), reverse saturation current \( I_0 \), and forward voltage \( V_F \).]
Example 2

- Consider a p-type semiconductor in direct contact with an n-type semiconductor. Sketch out as a function of position:
  a). Carrier density.
  b). Charge density.
  c). Electric field.
  d). Electrostatic Potential.
- Include labels for the positions for the edge of the depletion region in the p-type and n-type: \( w_p \), \( w_n \) semiconductors respectively.

Example 2

- This is something that is easier to just know, rather than derive from a thought experiment.
- We are asked to sketch the profiles.
- This means we are interested in the shape of the profiles; not their height.
- We have been asked for \( w_p \), \( w_n \) so include it.
Example 3

- You are told that the conductance of a diode ($g_d$) with a finite series resistance of $r_s$ is described by:

$$g_d = \frac{1}{n} \left( \frac{1}{q} - \frac{r_s g_d}{k_B T} \right)$$

- Where:
  - $I$ is the current flowing.
  - $q$ is the fundamental unit of charge.
  - $n$ is the ideality factor.
  - $k_B$ is the Boltzmann Constant.
  - $T$ is the temperature.

From the extrapolated data plotted below, approximate the contact resistance.

- Gradient = 1.93 A$^{-1}$
- Intercept = 20 S

- Assume the measurement is carried out at 300K and that $n = 1.$
Example 3

• First, re-arrange equation in the form of straight line:

\[ g_d = \frac{lq(1 - r_s g_d)}{nk_BT} \]

\[ \frac{g_d}{I} = -\frac{q r_s}{nk_BT} g_d + \frac{q}{nk_BT} \]

• Since \( g_d/I \) is plotted as a function of \( g_d \), we can immediately identify gradient as:

\[ m = -\frac{q r_s}{nk_BT} \]

Example 3

• We are told that \( m = 1.93 \ A^{-1} \).
• Re-arrange equation for the gradient:

\[ r_s = \frac{m n k_B T}{q} \]

• Put in numbers:

\[ r_s = \frac{1.93 \times 1 \times 1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \]

\[ r_s \approx 0.05 \Omega \]
Example 3

- Alternatively: we could look at the $x$-axis intercept.

\[
\frac{g_d}{I} = -\frac{qr_s}{nk_B T} g_d + \frac{q}{nk_B T}
\]

- If $g_d/I = 0$, then:

\[
-\frac{qr_s}{nk_B T} g_d + \frac{q}{nk_B T} = 0
\]

\[
\frac{q}{nk_B T} = \frac{qr_s}{nk_B T} g_d
\]

\[
1 = r_s g_d
\]

\[
r_s = \frac{1}{g_d}
\]

Example 3

- From the graph we are told that the $x$-intercept occurs at:

\[
g_d = 20 \text{ S}
\]

- Take the inverse to get the series resistance:

\[
r_s = \frac{1}{20}
\]

\[
r_s = 0.05 \Omega
\]
Example 1

• With the help of diagrams, describe the difference between photon absorption in a direct and an indirect bandgap semiconductor.

• Here you would be expected to draw simplified band diagrams.
Example 1

• In a **direct bandgap** semiconductor, a photon can be absorbed without the need for a transfer of momentum: an electron and hole are created **directly** from the photon.

• In an **indirect bandgap** semiconductor, conservation of moment requires that a transfer of momentum also occurs (e.g. via a phonon) for an electron to be excited from the top of the valence band to the bottom of the valence band.

Example 2

• The figure below shows current density ($J$) plotted as a function applied voltage ($V$) when a solar cell is illuminated with 100 mW/cm$^2$ light.

• If we are told that at the maximum power point, $J = 18.18$ mA/cm$^2$ and $V = 0.55$V, what is the efficiency of this solar cell?
Example 2

• We would be told the efficiency is given by:

\[ \eta = \frac{V_{oc}J_{sc}FF}{P} \]

• We are told the power density: \( P = 100 \text{ mW/cm}^2 \).
• We would be expected to know what is meant by open-circuit voltage, and short-circuit current, and how to read them off a graph.

\[ V_{oc} = 1.0 \text{V}. \]
\[ J_{sc} = 20 \text{ mA/cm}^2. \]

• We now need to evaluate the fill-factor:

\[ FF = \frac{V_{mp}J_{mp}}{V_{oc}J_{sc}} \]

• We are told the voltage and current density at the maximum power point:

\[ J_{mp} = 18.18 \text{ mA/cm}^2. \]
\[ V_{mp} = 0.55 \text{V}. \]
Example 2

• $V_{oc} = 1.0\text{V}$.  
• $J_{sc} = 20\text{ mA/cm}^2$.  
• Putting in the numbers:

$$FF = \frac{V_{mp}J_{mp}}{V_{oc}J_{sc}}$$

$$FF = \frac{0.55 \times 18.18}{1.0 \times 20}$$

$FF = 50\%$

• Now evaluate PCE:

$$\eta = \frac{V_{oc}J_{sc}FF}{P}$$

$$\eta = \frac{20 \times 1 \times 0.5}{100}$$

$\eta = 10\%$

Lectures 14 and 15
MOS Capacitors
Example 1

• Consider a Si/SiO\textsubscript{2} capacitor with a depletion width of \(W = 2\mu m\), and oxide thickness of \(t_{ox} = 10\) nm, a generation lifetime of \(\tau_g = 10\) \(\mu\)s, and an intrinsic carrier density of \(n_i = 10^{10}\) cm\(^{-3}\).

• Determine the minimum voltage sweep-rate which we could employ to ensure that inversion is observable in a capacitance-voltage measurement.

Example 1

• We would be given the following condition required for inversion to be observable in an \(CV\) measurement:

\[
\frac{dV}{dt} \leq \frac{q n_i W}{\tau_g C_{ox}}
\]

Capacitance / area (would be told this)

• This is basically an exercise in putting in numbers.

• We are given everything apart from the oxide capacitance, which is given by:

\[
C_{ox} = \frac{\varepsilon_0 \varepsilon_r}{t_{ox}}
\]
Example 1

- We would be told the vacuum permittivity and the relative permittivity of silicon oxide:
  - \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1} \).
  - \( \varepsilon_r = 4. \)
- We can therefore calculate the capacitance per unit area. Note \( \varepsilon_0 \) has units of Fm\(^{-1}\), so best stick to SI:
  \[
  C_{ox} = \frac{\varepsilon_0 \varepsilon_r}{t_{ox}}
  \]
  \[
  C_{ox} = \frac{8.85 \times 10^{-12} \times 4}{10 \times 10^{-9}}
  \]
  \[
  C_{ox} = 3.54 \times 10^{-3} \text{ Fm}^{-2}
  \]

Example 1

- Now we can just enter the rest of the numbers into our inequality:
  \[
  \frac{dV}{dt} \leq \frac{qn_i W}{\tau_g C_{ox}}
  \]
  \[
  \frac{dV}{dt} \leq \frac{1.60 \times 10^{-19} \times 10^{16} \times 2 \times 10^{-6}}{10 \times 10^{-6} \times 3.54 \times 10^{-3}}
  \]
  \[
  \frac{dV}{dt} \leq 0.0905 \text{ V/s}
  \]
- I.e. if we sweep the voltage slower than a rate of ~ 90 mV/s, we should observe inversion.
Example 1

- Sketch, roughly, the transfer characteristics of a p-type transistor. Label axis appropriately.
- A p-type transistor is one capable of injecting and transporting holes.
- It will only allow current to flow for negative biases.

![Diagram of Field Effect Transistor]
Example 2

- Describe, with diagrams, why the source-drain current in a transistor saturates when a drain voltage above a certain values is applied.
- This can be understood in terms of the direction of the electric field in the transistor channel.
- If the source-drain voltage is zero, or very small relative to the source gate voltage, then the electric field vector in the channel points ~down. The accumulated carriers are uniform, and the device is Ohmic ($I_{DS} \propto V_D$)

Example 2

- As the source-drain voltage increases, the field rotates and the distribution of carriers becomes non-uniform → voltage characteristics become non-uniform.
- Eventually the carriers are injected and swept away so quickly that the charge density near the drain electrode is ~zero. This is the pinch off point.
Example 2

- The region adjacent to the drain electrode is devoid of carriers → it has a very high resistance.

- We can consider it as resistors in series.

- As the depletion region increases in size, so does its resistance. Any extra voltage applied is dropped across this region → not effecting the device.

Lecture 17
Field-Effect Transistors 2
Example 1

• Consider a transistor channel of length $L$, width $W$ and thickness $D$, with an inhomogeneous carrier distribution: $n(x)$.

• By slicing the channel up into sections of length $\delta x$, we can approximate the carrier distribution as homogeneous in each slice: $n \neq n(x)$.

Example 1

• Using this formalism, derive an equation for drain current ($I_D$) as a function of drain voltage ($V_D$) and gate voltage ($V_G$) for a given channel length ($L$) and width ($W$), and channel capacitance per unit area ($C_{ox}$).
Example 1

- Here we are essentially following the procedure from Lecture 17 to derive the gradual channel approximation.
- See Lecture 17 for variable names if unclear.
- Start by considering the total number of carriers in the slice:
  \[ n(x) = \frac{N(x)}{V} = \frac{N(x)}{WD\delta x} \]
- Put back into our (given) equation for conductivity:
  \[ \sigma(x) = e\mu n(x) = \frac{e\mu N(x)}{WD\delta x} \]

Example 1

- We identify \( eN(x) \) as total charge: \( Q_{mob}'(x) = eN(x) \):
  \[ \sigma(x) = \frac{\mu Q_{mob}'(x)}{WD\delta x} \]
- We would be given the equation for 2-dimensional charge density as a function of position:
  \[ Q_{mob}(x) = C_{ox}(V_G - V_T - V(x)) \]
- We can write 2-dimensional charge density \( Q_{mob} \) in terms of total charge \( Q_{mob}' \)
  \[ Q_{mob}(x) = \frac{Q_{mob}'(x)}{W\delta x} \]
  \[ Q_{mob}'(x) = Q_{mob}(x)W\delta x \]
Example 1

• Substitute $Q_{mob}'(x)$ into our equation for $\sigma(x)$

$$\sigma(x) = \frac{\mu Q_{mob}(x)W \delta x}{WD \delta x}$$

$$\sigma(x) = \frac{\mu Q_{mob}(x)}{D}$$

• We would be given the equation (from Lecture 2) for resistivity:

$$\rho = \frac{RA}{L}$$

resistivity, resistance
resistor length
resistor cross-sectional area

• In our system we identify the cross-sectional area as: $WD$.

• Identify length as $\delta x$.

$$\sigma = \frac{1}{\rho} = \frac{\delta x}{RWD} \quad \delta R = \frac{\delta x}{\sigma(x)WD}$$

• Note: We use $\delta R$ because we are talking about a small part of the channel, rather than the resistance ($R$) of the whole channel.

• Substitute in $\sigma(x)$:

$$\delta R = \frac{D \delta x}{\mu Q_{mob}(x)WD} \quad \delta R = \frac{\delta x}{\mu Q_{mob}(x)W}$$
Example 1

\[ \delta R = \frac{\delta x}{\mu Q_{mob}(x)W} \]

- Now consider what happens to the change in resistance as \( \delta x \to 0 \):

\[ dR = \frac{dx}{\mu Q_{mob}(x)W} \]

- We can use the differential form of Ohm’s law to describe the current arising from changes in resistance with respect changes in voltage:

\[ I_D = \frac{dV}{dR} \]

Example 1

- We call the current flowing in our device the source-drain current:

\[ I_D = \frac{dV}{dR} \quad dR = \frac{dx}{\mu Q_{mob}(x)W} \]

- Combine these two differential equations:

\[ I_D = \frac{dV}{dx} \mu Q_{mob}(x)W \]

- Substitute in our equation for \( Q_{mob}(x) \):

\[ I_D = \frac{dV}{dx} \mu C_{ox}(V_G - V_T - V(x))W \]
Example 1

\[ I_D = \frac{dV}{dx} \mu C_{ox} (V_G - V_T - V(x)) W \]

- Separate the differentials:
  \[ I_D dx = \mu C_{ox} (V_G - V_T - V(x)) W dV \]
- Which we can evaluate by integration:
  \[ \int \, I_d dx = \int \, \mu C_{ox} (V_G - V_T - V(x)) W dV \]
- We just need to chose the limits.

Example 1

- Look at our whole channel again:

  \[ V(0) = 0 \quad V(L) = V_D \]
- We need to integrate over the whole channel.
- So we need to integrate from \( x = 0 \) (where \( V(x) = 0 \)) to \( x = L \) (where \( V(x) = V_D \)).
Example 1

• Put in these limits:

\[
\int_0^L I_D dx = \int_0^{V_D} \mu C_{ox} (V_G - V_T - V(x)) W dV
\]

• Current does not vary with position, so:

\[
\int_0^L I_D dx = [I_D x]_0^L = I_D (L - 0) = I_D L
\]

• Hence we can say:

\[
I_D = \frac{W}{L} \mu C_{ox} \int_0^{V_D} (V_G - V_T - V(x)) dV
\]

Example 1

\[
I_D = \frac{W}{L} \mu C_{ox} \int_0^{V_D} (V_G - V_T - V(x)) dV
\]

• \(V_G\) and \(V_T\) are just constants. So:

\[
I_D = \frac{W}{L} \mu C_{ox} \left[ V_G V - V_T V - \frac{V^2}{2} \right]_0^{V_D}
\]

\[
I_D = \frac{W}{L} \mu C_{ox} \left( V_G V_D - V_T V_D - \frac{V_D^2}{2} \right)
\]

\[
I_D = \frac{W}{L} \mu C_{ox} \left( (V_G - V_T)V_D - \frac{V_D^2}{2} \right)
\]
Example 2

• The figure (top) shows the transfer characteristics of a field effect transistor (FET). The figure below shows the same data differentiated. Use this data to approximate the field-effect mobility in the linear regime. The FET has a length and width of $L = 10 \ \mu m$ and $W = 1000 \ \mu m$ respectively, with an areal capacitance of $C_{ox} = 200 \ \text{nF/cm}^2$. We are told $V_T = -7V$ and $V_D = 10V$.

• We would be either be given the below equation, or expected to derive it from the gradual channel approximation (result from last problem).

$$\mu_{\text{lin}} = \frac{L}{W C_{ox} V_D} \frac{dI_D}{dV_G}$$

• For the linear-regime approximation to be valid we require: $|V_D| \ll |V_G - V_T|$. 

• We are told $V_T = -7V$, so as long $V_G \gg 3V$, we are ok.
Example 2

- From this figure we would hence say:
  \[
  \frac{dI_D}{dV_G} \approx 1 \times 10^{-6} \text{A/V}
  \]

- The rest of the question is entering numbers:

  \[
  \mu_{\text{lin}} = \frac{L}{W C_{ox} V_D} \frac{dI_D}{dV_G}
  \]

  \[
  \mu_{\text{lin}} = \frac{10}{1000 \times 200 \times 10^{-9} \times 10} \times 1 \times 10^{-6} = 50 \text{cm}^2/\text{Vs}
  \]
Example 1

- The figure below shows (idealized) output characteristics.

- From this data approximate the output-conductance in the linear regime.

\[ g_d = \frac{\partial I_D}{\partial V_D} \]

\[ g_d = \frac{10 \times 10^{-6}}{20} = 5 \times 10^{-7} \text{ A/V} \]
Example 2

- Under what circumstances would it be more appropriate to report transconductance rather than field-effect mobility?
- In situations when applying the gradual channel approximation (GCA) to current-voltage data may be invalid.
- For example:
  - Source-gated devices.
  - Devices with significant contact-resistance.
  - Materials known to have field-dependent mobility.

Example 3

- The figure shows the transfer characteristics of a transistor plotted on a linear-linear scale, extrapolated in the linear regime to $I_D = 0$.
- Using this data, and the equation below, approximate the threshold voltage for this transistor.

$$I_D = \frac{W}{L} \mu C_{ox} \left[ V_G - V_T - \frac{V_D}{2} \right] V_D$$
Example 3

• Here we just have to consider what happens when $I_D=0$:

$$\frac{W}{L} \mu C_{ox} \left[ V_G - V_T - \frac{V_D}{2} \right] V_D = 0$$

$$V_G - V_T - \frac{V_D}{2} = 0 \quad \Rightarrow \quad V_T = V_G - \frac{V_D}{2}$$

• When $I_D = 0$V, we observe $V_G = 5$V. We are also told that $V_D = 10$V.

$$V_T = 5 - \frac{10}{2} = 0\text{V}$$
Example 1

• Describe why we would be interested in carrying out contactless measurements.
  • The interfaces between contacts and semiconductors is complex. If the interfaces are not optimized (e.g. if energy offsets are not aligned), then electrical characteristics may not be representative of the material.
  • We may want to know its properties as a material itself, rather than a device.
Example 2

• The figure below shows the reflectivity of a TRMC cavity and it’s dimensions. From these figure approximate the sensitivity factor of the cavity. Assume the cavity is in air.

\[ D = 10.16\text{mm} \]
\[ W = 22.86\text{mm} \]
\[ L = 76.20\text{mm} \]

\[ L = 7.62\text{cm}. \]
\[ W = 2.286\text{cm}. \]
\[ D = 1.016\text{cm}. \]
\[ \beta = 2.25. \]

The equation for the sensitivity factor in the cavity would be given (along with labeled variables):

\[
K = \frac{2Q \left(1 + \frac{1}{\sqrt{R_0}}\right)}{\pi f_0 \varepsilon_0 \varepsilon_r L \beta}
\]
Example 2

- The parameters $f_0$, $R_0$ and $Q$ can be extracted from the reflectance vs frequency plot.

\[ f_0 \approx 9.03 \text{ GHz} \]

\[ R_0 \approx 0.03 \]

\[ \Delta W \approx 3.8 \text{ MHz} \]

\[ Q = \frac{f_0}{\Delta W} \]

- $\Delta W$: Full-width at half-maximum (FWHM) of resonance.
- Hence we would say:
  - $f_0 = 9.03$ GHz.
  - $R_0 = 0.03$.
  - $\Delta W = 3.8$ MHz.
  - $Q = 2376$.  

\[ Q \approx 0.8 \]
Example 2

- We would be given $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm$^{-1}$.
- We would be expected to know that $\varepsilon_r = 1$ for air.
- The rest of the question would just be entering values:

$$K = \frac{2Q \left( 1 + \frac{1}{\sqrt{R_0}} \right)}{\pi f_0 \varepsilon_0 \varepsilon_r L \beta}$$

$$K = \frac{2 \times 2376 \times \left( 1 + \frac{1}{\sqrt{0.03}} \right)}{\pi \times 9.03 \times 10^9 \times 8.85 \times 10^{-12} \times 1 \times 0.0762 \times 2.25}$$

$$K = 7.48 \times 10^5 \Omega$$

Example 3

- The figure shows measured photoconductance as a function of time, in a time-resolved microwave conductivity experiment.
- If we are told:
  - The incident fluence is $2 \times 10^{13}$ cm$^{-2}$.
  - The fractional absorbance at the excitation (laser) wavelength is 55%.
  - $\beta$ is 2.25.
  - Determine $\phi \Sigma \mu$. 
Example 3
• We would be given:

\[ \phi \Sigma \mu = \frac{\Delta G_{\text{max}}}{\beta e I_0 F_A} \]

• All we have to do is identify the peak value, then enter numbers.

\[ \phi \Sigma \mu = \frac{2.8 \times 10^{-6}}{2.25 \times 1.6 \times 10^{-19} \times 2 \times 10^{11} \times 0.55} \]

\[ \phi \Sigma \mu = 7.1 \text{ cm}^2/\text{Vs} \]

Example 4
• If we are carrying out a time resolved conductivity measurement and are observing the time is limited by the response time of the cavity, what action can be taken?

• The response time \( \tau_{RC} \) of the cavity is described by:

\[ \tau_{RC} = \frac{Q}{\pi f_0} \]

• \( Q \): Quality factor of the cavity.
• \( f_0 \): Resonance frequency of the cavity.
Example 4

- To reduce the response time you can either reduce the quality factor → increase the resonance width.
- Or increase the frequency.
Example 1

- A weekly-absorbing, non-magnetic material has a relative permittivity of $\varepsilon_r = 4$. If light enters this material, from air, with an incident angle of $\theta = 30^\circ$, determine the angle the light makes to the surface normal in the material. Assume a perfectly flat interface with no reflection.
- We would just describe the situation as shown.
- $\theta = \theta_1 = 30^\circ$.
- $n_1 = 1$ (air).

![Image of light propagation through two mediums](image)

Example 1

- We would be given the following equation describing the refractive index of weekly absorbing materials:

$$n = \sqrt{\varepsilon_r \mu_r}$$

- $\varepsilon_r$ is the relative permittivity.
- $\mu_r$ is the relative permeability.
- For non-ferromagnetic materials $\mu_r \approx 1$.
- Hence for our material:

$$n = \sqrt{\varepsilon_r} = \sqrt{4} = 2$$
Example 1

- We can then just use Snell’s Law:
  \[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
- We seek \( \theta_2 \):
  \[ \theta_2 = \arcsin \left( \frac{n_1 \sin \theta_1}{n_2} \right) \]
- Enter values:
  \[ \theta_2 = \arcsin \left( \frac{1 \times \sin(30)}{2} \right) \]
  \[ \theta_2 = \arcsin \left( \frac{1}{4} \right) \]
  \[ \theta_2 = 14.48^\circ \]

Example 2

- Describe using diagrams what is meant by an exciton.
- In certain materials, instead of a free electron-hole pair being produced by incident light, the electron-hole pair are produced in a bound state:

  Free Carriers

  ![Free Carriers Diagram](image)

  Exciton

  ![Exciton Diagram](image)
Example 2

- Excitons are:
  - A quasi-particle that exists only in solids.
  - Electron and hole remain bound.
  - They are better described by a single wavefunction than separate wavefunction.
  - Generated by lower energy photons than band-to-band transitions.

Example 2

- The physical description is as follows:
  - Light generates an electron and hole.
  - They separate (conservation of momentum).
  - Positively charged hole distorts electrons in lattice.
  - This provides screening / repulsion to free electron.
  - Equilibrium occurs with a binding energy \( E_B \).
  - Thermal energy \( k_B T \) required to collapse charges.
Example 3

• In far-field optical-microscopy, we are attempting to image a sample using green ($\lambda = 525\text{nm}$) light in air.
• The lens is 4 times its diameter ($d$) away from the focal point of sample:

```
\[ \begin{array}{c}
\text{lens} \\
\uparrow \\
\downarrow \\
\text{focal point}
\end{array} \]
```

• What is the resolution and depth of field for this microscope?

```
\[ S = \frac{0.61\lambda}{NA} \quad D = \frac{\sqrt{n^2 - NA^2}}{NA^2} \lambda \]
```

• $NA =$ numerical aperture $= n\sin \theta$
• $n =$ refractive index.
• $\theta =$ acceptance angle of the microscope.
Example 3

- The tricky part of the question is evaluating $\theta$
- We can describe the focal point in relation to the lens as a triangle:

\[
\frac{d}{4d} = \frac{1}{8} \quad \theta = \arctan\left(\frac{1}{8}\right) \quad \theta = 7.12^\circ
\]

- Now we just have to enter values.
- We have established, $\theta = 7.12^\circ$

\[
NA = n \sin \theta = 1 \times \sin 7.12 = 0.124
\]

- $S$ and $D$ can then be evaluated:

\[
S = \frac{0.61 \lambda}{NA} = \frac{0.61 \times 525 \times 10^{-9}}{0.124} = 2.58 \, \mu m
\]

\[
D = \frac{\sqrt{n^2 - NA^2}}{NA} \lambda = \frac{\sqrt{1 - 0.124^2}}{0.124} \times 525 \times 10^{-9}
\]

\[
D = 34 \, \mu m
\]
Example 1

- If we accelerate electrons with a 200kV source, what is the difference in wavelength calculated with and without considering special relativity?
- Start with our non-relativistic equation (would be given):

\[ \lambda_{nr} = \frac{h}{\sqrt{2m_0eV}} \]

- \( \lambda \) = Electron wavelength.
- \( h \) = Planck Constant.
- \( e \) = Charge on electron.
- \( V \) = Accelerating voltage.
- \( m_0 \) = Rest mass of electron.
Example 1

- \( V = 200 \text{ kV} \) is the only parameter not a fundamental constant:

\[
\lambda_{nr} = \frac{6.63 \times 10^{-34}}{\sqrt{2} \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19} \times 2 \times 10^{5}}
\]

\[
\lambda_{nr} = 2.74 \times 10^{-12} \text{ m}
\]

\[
\lambda_{nr} = 2.74 \text{ pm}
\]

- Now consider relativistic equation (also given):

\[
\lambda = \frac{h}{\sqrt{2m_0eV}} \sqrt{\frac{1}{1 + \frac{eV}{2m_0c^2}}}
\]

\[r\text{ for } \text{“relativistic”}\]

- We could just write this as:

\[
\lambda_r = \lambda_{nr} \frac{1}{\sqrt{1 + \frac{eV}{2m_0c^2}}}
\]

- Now enter values:

\[
\lambda_r = 2.74 \times 10^{-12} \times \frac{1}{\sqrt{1 + \frac{1.602 \times 10^{-19} \times 2 \times 10^{5}}{2 \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19} \times (3 \times 10^8)^2}}}
\]

\[
\lambda_r = 2.74 \times 10^{-12}/1.09
\]

\[
\lambda_r = 2.51 \times 10^{-12} \text{ m}
\]

\[
\lambda_r = 2.51 \text{ pm}
\]
Example 1

- So the results are:
  \[ \lambda_{nr} = 2.74 \text{pm} \]
  \[ \lambda_r = 2.51 \text{pm} \]

- And the difference is:
  \[ \Delta \lambda = \lambda_{nr} - \lambda_r \]
  \[ \Delta \lambda = 2.74 - 2.51 \]
  \[ \Delta \lambda = 0.23 \text{pm} \]

- This is small, but not zero.

Example 2

- Describe briefly, the operating principle of a scanning electron microscope.
  - Electrons are generated with an electron gun. Either this is a thermionic source or a high energy field-emission source.
  - Electrons are accelerated towards a sample (under vacuum) by means of a positively charged anode.
  - A set of electromagnetic lenses are used to focus the beam to a small point on the sample surface.
  - The focused beam is then deflected using electromagnetic deflection coils.
Example 2

- Electrons hit the sample surface at a particular point and emit secondary electrons.
- More electrons are emitted by higher parts of the sample. Those that are close to the focal point of the beam.
- The secondary electrons are attracted towards a positively charged secondary electron detector.
- The detector then converts the counted electrons into a pixel.
- More electrons detected → brighter image.
- Surface topography is the main source of contrast in an SEM.

Example 3

- List some of the limitations of transmission electron microscopy.
  - Samples must be very thin.
  - Sample will often have to be cut with a focused ion beam before a measurement can take place.
  - Transmission electron microscopes are expensive and require highly-trained technicians to operate.
  - A high vacuum is required.
Example 4

• Describe the Auger process and how it is exploited in Auger electron spectroscopy (AES).

  • High energy electrons are incident on the sample.
  • These high energy electrons can cause core level electrons (e.g. $n=1/\text{K}$) to be ejected from the atom.
  • Higher energy atoms will then fall to the low energy state to fill the hole.
  • This process can result in a large amount of excess energy; sufficient to cause another electron to be emitted from the atom.

• The energy of this emitted (Auger) electron can then be measured in the AES experiment.

  • It turns out that the energy involved in the transition from the shallower- to deep-electron state is unique to each atom.
  • Therefore the measured energy of the emitted electrons can be used to identify the atoms in the sample.
Lecture 22
Ion Beam Techniques

Example 1

• Briefly describe what is meant by the term sputtering, and how it can be useful in ion-based spectroscopy.
  • Sputtering occurs when ions impinge upon a sample surface and transfer enough momentum into the lattice to dislodge atoms, molecules, or ions from the near-surface region.
  • Sputtering is described as a collision cascade, in which primary ion hits the surface, losing momentum and energy via a series of elastic and inelastic scattering events with target atoms.
Example 1

- In turn, some of the target atoms are dislodged by recoil and then displace other atoms.
- Our aim is to detect these dislodged atoms / molecules and use this information to say something about the composition of our sample.

Example 2

- We are carrying out a Time-of-flight secondary ion mass spectrometry (ToF SIMS) experiment.
- In the simplest form of the experiment we measure the time ($t$) it takes our emitted ions to travel a distance $L$ to the detector accelerated due to a reflector voltage $V$.
- We conduct a SIMS measurement with a reflector voltage of $V = 85V$ and a travel distance $L = 10$ cm.
- We are here just looking at data from the surface (i.e. at one particular depth, and before we have milled the sample).
Example 2

• For a certain peak in the SIMS raw spectrum we measure a transit time of 2.7 μs. If assume that all ions have a single charge (i.e. +e), use the table below to determine the element we are detecting.

<table>
<thead>
<tr>
<th>Element</th>
<th>Mass (amu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>1</td>
</tr>
<tr>
<td>Boron</td>
<td>10.8</td>
</tr>
<tr>
<td>Carbon</td>
<td>12</td>
</tr>
<tr>
<td>Silicon</td>
<td>28</td>
</tr>
<tr>
<td>Lead</td>
<td>207</td>
</tr>
</tbody>
</table>

• You would be told that 1 amu = 1.66 × 10^{-27} kg.

Example 2

• You would be told the kinetic energy \( KE \) and potential energy \( PE \) of an ion with a velocity \( v \) and under an applied voltage of \( V \) can be described by:

\[
KE = \frac{1}{2}mv^2 \quad \quad \quad PE = qV
\]

• We can then describe the velocity in terms of the transit distance \( L \) and transit time \( t \):

\[
\text{You would not be given} \quad v = \frac{L}{t}
\]

\[
KE = \frac{mL^2}{t^2}
\]
Example 2

- Equating kinetic and potential energy, we can say;

\[ KE = PE = \frac{1}{2}mv^2 = qV \quad \rightarrow \quad \frac{m}{q} = \frac{2Vt^2}{L^2} \]

- We now have enough information to evaluate the mass-to-charge ratio:

\[ \frac{m}{q} = \frac{2 \times 85 \times (2.7 \times 10^{-6})^2}{0.1^2} = 1.24 \times 10^{-7} \text{kg/C} \]

- We just need to convert this into a value of amu/e rather than kg/C.

Example 2

\[ \frac{m}{q} = 1.24 \times 10^{-7} \text{kg/C} \]

- Let’s do this in 2 steps. First convert to amu / C:

\[ \frac{m}{q} = 1.24 \times 10^{-7} \times \frac{1}{1.66 \times 10^{-27}} = 7.49 \times 10^{19} \text{amu/C} \]

- Now convert to amu/e (you would be given \( e \)).

\[ \frac{m}{q} = 7.49 \times 10^{19} \times 1.6 \times 10^{-19} = 12 \text{amu/e} \]

- I.e. from the table we can say we have detected carbon.
Lecture 23
X-Ray & UV Techniques

Example 1
• Briefly describe the differences between X-Ray Photoelectron Spectroscopy (XPS) and Ultraviolet Photoelectron Spectroscopy (UPS).
  • The techniques are very similar in practice. The main experimental difference is the wavelength / energy of the incident photons. XPS uses X-rays, UPS uses ultraviolet photons.
  • The higher energy photons of XPS cause photoelectron emission from core levels.
  • The lower energy photons of UPS cause photoelectron emission from valence levels.
Example 1

- These differences mean that we are probing elemental composition and bonding properties with XPS, and the energy of band with UPS.

Example 2

- The figure to the right shows the experimental X-ray diffraction pattern of a thin film of a known material (black). Also, shown are the calculated diffraction patterns along known crystal axis.
- From this data, determine the primary orientation of the film.
Example 2

- The closest match to the experimental data is the calculated diffraction pattern labeled [100].
- Therefore we can conclude that the film is primarily orientated along the [100] crystal axis.

Good Luck!