Lecture 8
Carrier Lifetime
Schroder: Chapter 7

Announcements
Homework 1/5:
• Due Now.
• I will return it on 22nd April.
Homework 2/5:
• Will be posted online after class.
• Due Friday 24th April at the start of the lecture (09:00am).
• I will return it one week later (1st May).
Lecture 8

• Lifetime Overview.
• Shockley-Read-Hall Recombination.
• Radiative Recombination.
• Auger Recombination.
• Lifetime Revisited.

Lifetime Overview
Lifetime

• In electronic devices we inject carriers and typically expect them to travel as far as possible.

• In an idealized situation we expect every carrier to reach the opposite electrode and be extracted.

• Today’s lecture is concerned with the fact that this is not the case.

Lifetime

• Carriers that are being transported can recombine while in a device.

• The average time that carriers last is referred to as the lifetime.
**Lifetime**

- Values of recombination and lifetime **heavily** dependent on the situation.
- They are probably one of the most ambiguous parameters in the field.
- Lifetime depends on:
  - Material type.
  - Carrier density.
  - Temperature.
  - Illumination density.
  - Point defect density.
  - Device Type.
  - Surface properties.
  - Grain boundaries.
  - Bias conditions.
  - Many others...

**Recombination Lifetime**

- In this lecture we provide a simplified overview.
- According to Schroder there are two types of lifetime:

  - **Recombination Lifetime**
  - **Generation Lifetime**

- Generation lifetime is basically the time it takes to thermally create carriers. We will not study it today.
Recombination Lifetime

- Today we are talking about electronic devices here (mainly transistors).
- In this context we are talking about the lifetime of minority carriers in a sea of majority carriers.
- For example, we are concerned with the lifetime of electrons in a p-type semiconductor.

Aside: Solar Cells

- Terminology depends on device type.
- In solar cells carrier lifetime is very important.
- Both holes and electrons need enough time to make it to electrodes.
- Lifetime in the context of solar cells is normally specified by carrier type: $\tau_e$ or $\tau_h$.
- If the term “lifetime” is used without specifying a carrier type it will refer to the carrier with the longest lifetime.
**Lifetime**

- But today we are talking about the recombination lifetime ($\tau_r$) of minority carriers in electronic devices.
- We will typically talk about electrons in a p-type semiconductor, but the concepts are identical to the opposite situation.
- The recombination lifetime is separated into three terms:

$$\frac{1}{\tau_r} = \frac{1}{\tau_{SRH}} + \frac{1}{\tau_{rad}} + \frac{1}{\tau_{Auger}}$$

**Lifetime**

- A convenient way to visualize these three processes is to consider the energy band diagram.
- Consider how energy is conserved in each case.
Lifetime

- We can also consider the number of charge-carriers involved in the process.

\[
\begin{align*}
\text{~SRH (monomolecular)} & \quad \text{Radiative (bimolecular)} & \quad \text{Auger (trimolecular)} \\
\text{Carrier} & \quad \text{Hole} & \quad \text{Hole} \\
\text{Trap} & \quad \text{Electron} & \quad \text{Electron}
\end{align*}
\]

Shockley-Read-Hall Recombination
SRH Recombination

- There are four types of trap emission and capture processes. Same notation as Lecture 6.

Net Capture Rate

- The net rate of electron capture into a trap, $U_n$, is given by the rate of electron trap capture minus the rate of electron trap emission:

$$U_n = r_a - r_b$$

- Recall from Lecture 6 we can describe the electron capture rate by:

$$r_a = \sigma_n v_{th} n N_T (1 - f_T)$$

- And the electron emission rate by:

$$r_b = e_n N_T f_T$$
Parameters

\[ r_a = \sigma_n v_{th}^e n N_T (1 - f_T) \quad r_b = e_n N_T f_T \]

- \( r_a \) = Rate of electron capture.
- \( \sigma_n \) = Electron capture cross-section (e.g. cm\(^2\)).
- \( v_{th}^e \) = Electron thermal velocity.
- \( n \) = Delocalized electron density in conduction band.
- \( N_T (1 - f_T) \) = Number density of empty traps.
- \( r_b \) = Rate of electron emission.
- \( e_n \) = Emission coefficient for electrons
- \( N_T f_T \) = Number density of filled traps.

e\(^-\) Emission coefficient

- We also know from Lecture 6, that we can describe the emission coefficient in terms of temperature and band energies:

\[ e_n = \sigma_n v_{th}^e N_c \exp \left( - \frac{E_C - E_T}{k_B T} \right) \]

- \( N_c \) = Number density conduction band states.
- \( E_C \) = Conduction band energy.
- \( E_T \) = Trap state energy.
- \( k_B \) = Boltzmann Constant.
- \( T \) = Temperature.
Net Capture Rate

• Substituting this back into our equation for net capture rate:

\[ r_a = \sigma_n v_{th} n N_T (1 - f_T) \]
\[ r_b = e_n N_T f_T \]

\[ U_n = r_a - r_b = \sigma_n v_{th} n N_T (1 - f_T) - e_n N_T f_T \]

\[ e_n = \sigma_n v_{th} N_c \exp \left( -\frac{E_C - E_T}{k_b T} \right) \]

\[ U_n = \sigma_n v_{th} \left[ n N_T (1 - f_T) - N_c \exp \left( -\frac{E_C - E_T}{k_b T} \right) N_T f_T \right] \]
**Net Capture Rate - Holes**

- We can (but won’t) derive a similar expression for the net capture rate of holes:
  \[ U_p = \sigma_p v_{th}^h N_T[p f_T - p_1 (1 - f_T)] \]
  - \( \sigma_p \) = Hole capture cross-section (e.g. \( \text{cm}^2 \)).
  - \( v_{th}^h \) = Hole thermal velocity.
  - \( p \) = Delocalized hole density in valence band.
  - \( p_1 \) = concentration of holes in the valence band when the Fermi level is positioned at the trap energy level.

**Steady-State Trap Occupancy**

- Consider the *steady-state* situation in which the net rates of electron and hole capture are equal:
  \[ U_n = U_p \]
  - Under these conditions we can define the steady-state trap occupancy: \( f_T \rightarrow f_T^{(ss)} \).
  - This turns out to be:
    \[ f_T^{(ss)} = \frac{\sigma_n n + \sigma_p p_1}{\sigma_n (n + n_1) + \sigma_p (p + p_1)} \]
  - Where we have equated hole and electron thermal velocities:
    \[ v_{th}^e = v_{th}^h = v_{th} \]
SRH Capture Rate

- We define the Shockley-Read-Hall capture rate \( U_{SRH} \) as the steady-state situation in which the net rates of electron and hole capture are equal:

\[
U_n = U_p = U_{SRH}
\]

- It turns out this can be expressed as:

\[
U_{SRH} = \frac{\sigma_n \sigma_p v_{th} N_T (pn - n_i^2)}{\sigma_n (n + n_1) + \sigma_p (p + p_1)}
\]

- This is a very important relationship which is used extensively in processes involving SRH generation and recombination.

Excess Carrier Concentration

- If we go back to our steady-state trap occupancy factor:

\[
f_T^{(ss)} = \frac{\sigma_n n + \sigma_p p_1}{\sigma_n (n + n_1) + \sigma_p (p + p_1)}
\]

- We identify that this is not equal to the Fermi-Dirac distribution:

\[
f_T^{(ss)} \neq \frac{1}{1 + \exp\left(\frac{E_T - E_F}{k_B T}\right)}
\]

- \( f_T^{(ss)} \) is a non-equilibrium occupancy.
- Because carriers are constantly being trapped and detrapped, they are not Fermi-distributed.
Excess Carrier Concentration

- We can describe our non-equilibrium carrier distributions as perturbations from equilibrium (i.e. without the presence of the trap):

  \[ n = n_0 + \Delta n \quad \text{and} \quad p = p_0 + \Delta p \]

- \( n \) = Non-equilibrium electron density.
- \( p \) = Non-equilibrium hole density.
- \( n_0 \) = Equilibrium electron density.
- \( p_0 \) = Equilibrium hole density.
- \( \Delta n \) = Excess electron density.
- \( \Delta p \) = Excess hole density.

This formulation allows us to re-write \( U_{SRH} \).

\[ U_{SRH} = \frac{\sigma_n \sigma_p v_{th} N_T (pn - n_i^2)}{\sigma_n (n + n_1) + \sigma_p (p + p_1)} \]

- Start by enforcing charge neutrality on our excess carriers:

  \[ \Delta n = \Delta p \]

- Turns out this is a valid assumption as long as one of: \( n_0, p_0, n_1, p_1 \) is large compared to \( N_T \).
- See the Schroder or W. Shockley and W.E. Read, Phys. Rev. 87, 835 (1952) for why this is the case.
Excess Carrier Concentration

- Look at \( p_n - n_i^2 \) term of \( U_{SRH} \):
  \[
  p_n - n_i^2 = (n_0 + \Delta n)(p_0 + \Delta p) - n_i^2
  \]
  \[
  p_n - n_i^2 = n_0p_0 + n_0\Delta p + \Delta np_0 + \Delta n\Delta p - n_i^2
  \]
- Identify the equilibrium carrier density as the intrinsic carrier density:
  \[
  n_0p_0 = n_i^2
  \]
  \[
  p_n - n_i^2 = n_0\Delta p + \Delta np_0 + \Delta n\Delta p
  \]
- Enforce \( \Delta n = \Delta p \):
  \[
  p_n - n_i^2 = n_0\Delta n + \Delta np_0 + \Delta n\Delta n
  \]
  \[
  p_n - n_i^2 = (n_0 + p_0 + \Delta n)\Delta n
  \]

SRH lifetime

- Substitute \( p_n - n_i^2 \) back into \( U_{SRH} \):
  \[
  U_{SRH} = \frac{\sigma_n\sigma_p v_{th} N_T (p_n - n_i^2)}{\sigma_n(n + n_1) + \sigma_p(p + p_1)}
  \]
  \[
  U_{SRH} = \frac{\sigma_n\sigma_p v_{th} N_T \Delta n(n_0 + p_0 + \Delta n)}{\sigma_n(n + n_1) + \sigma_p(p + p_1)}
  \]
- And define the SRH lifetime:
  \[
  \tau_{SRH} = \frac{\tau_p(n_0 + n_1 + \Delta n) + \tau_n(p_0 + p_1 + \Delta p)}{p_0 + n_0 + \Delta n}
  \]
  \[
  \tau_p = \frac{1}{\sigma_p v_{th} N_T}
  \]
  \[
  \tau_n = \frac{1}{\sigma_n v_{th} N_T}
  \]
  \[
  U_{SRH} = \frac{\Delta n}{\tau_{SRH}}
  \]
**Important Limits**

- **Low-Level Injection:**
  - The excess minority carrier concentration is much less than the equilibrium (intrinsic) majority carrier concentration.
  - For p-type material:
    
    \[ \Delta n \ll p_0 \]

    \[ \tau_{SRH}^{(ll)} \approx \tau_n \]

  - For n-type material:
    
    \[ \Delta p \ll n_0 \]

    \[ \tau_{SRH}^{(ll)} \approx \tau_p \]

- **Very High Level Injection:**
  - Although this approximation is extensively employed for low-level injection conditions, note that it is only valid when the material is strongly of one type (e.g., when \( p_0 \gg n_0 \) for a p-type semiconductor) and when the trap location is sufficiently deep in the bandgap (e.g., when \( p_0 \gg p_1 \) and \( p_0 \gg n_1 \)).
  - For p-type material:
    
    \[ \Delta n \gg p_0 \]

    \[ \tau_{SRH}^{(hl)} \approx \tau_n + \tau_p \]

  - For n-type material:
    
    \[ \Delta p \gg n_0 \]

    \[ \tau_{SRH}^{(hl)} \approx \tau_n + \tau_p \]
Radiative Recombination

Overview

- Sometimes called bimolecular recombination.
- Basically just a hole and electron annihilating:

Energy Picture

Spatial Picture

\[ h \nu \]
Radiative Recombination

- This is an important process for a direct bandgap semiconductor, but is usually of no importance in an indirect semiconductor, unless it is extraordinarily pure.

- In an indirect semiconductor, a transfer of momentum is required for recombination (and generation).

\[ \nu = \frac{E_G}{h} \]

\(\nu\) ≠ \(\nu\)

frequency

Planck Constant

Band Gap

“nu” ≠ \(\nu\)
Recombination Rate

- The rate of radiative recombination rate \( R_{\text{rad}} \) is proportional to a product of the non-equilibrium carrier densities
  \[
  R_{\text{rad}} = \alpha n p
  \]
- It has units cm\(^{-3}\)s\(^{-1}\) (c.f. s\(^{-1}\) for SRH rate).
- We can show (but won’t) that:
  \[
  \alpha = \frac{R}{n_i^2}
  \]

- \( R \) is the recombination rate in thermal equilibrium.

Recombination Rate

\[
R = \frac{32\pi^2 c^3 (k_B T)^4}{h^4} \int_0^\infty \frac{k\hat{n}^3 x^3}{e^x - 1} \, dx
\]

- \( R \) is the recombination rate in thermal equilibrium.
- \( c \) is the speed of light in a vacuum.
- \( k_B \) is the Boltzmann Constant.
- \( h \) is the Planck Constant.
- \( k \) is the imaginary part of the index of refraction (i.e., the extinction coefficient)
- \( \hat{n} \) is the index of refraction (i.e., the real part of the complex index of refraction).
Recombination Rate

\[ R = \frac{32\pi^2 c^3 (k_B T)^4}{h^4} \int_0^\infty \frac{k_B T n^3 x^3}{e^x - 1} \, dx \]

- Where:
  \[ x = \frac{h\nu}{k_B T} \]

- \( \nu \): Photon frequency.
- So \( x \) is just optical energy normalized for thermal energy.
- \( R \) depends on the optical properties of the material (i.e., \( n \) and \( k \)) and on the temperature.

Replacing \( \alpha \) with \( R \) in the previous expression for the radiative recombination rate gives:

\[ R_{\text{rad}} = \alpha n p \]

\[ \alpha = \frac{R}{n_i^2} \]

Replacing nonequilibrium carrier concentrations by their equivalent excess carrier concentration expressions can get this into a more useful form:

\[ n = n_0 + \Delta n \]

\[ p = p_0 + \Delta p \]
Recombination Rate

\[ n = n_0 + \Delta n \quad p = p_0 + \Delta p \]

\[ R_{\text{rad}} = \frac{R}{n_i^2} np \]

\[ R_{\text{rad}} = \frac{R}{n_i^2} (n_0 + \Delta n)(p_0 + \Delta p) \]

\[ R_{\text{rad}} = \frac{R}{n_i^2} (n_0p_0 + n_0\Delta p + p_0\Delta n + \Delta n\Delta p) \]

- Again, use \( n_i^2 = n_0p_0 \):

\[ R_{\text{rad}} = R + \frac{R}{n_i^2} (n_0\Delta p + p_0\Delta n + \Delta n\Delta p) \]

Net Rate of Recombination

- The net rate of radiative recombination is defined as:

\[ U_{\text{rad}} = R_{\text{rad}} - R = \frac{R}{n_i^2} (n_0\Delta p + p_0\Delta n + \Delta n\Delta p) \]

- This is the most general solution of radiative recombination.
- To make further progress we enforce charge neutrality: \( \Delta n = \Delta p \).

- Where:

\[ \tau_{\text{rad}} = \frac{1}{B(p_0 + n_0 + \Delta n)} \]

\[ B = \frac{R}{n_i^2} \]

ECE / CHE 613 – Electronic Materials Characterization
Spring 2020 - John Labram
Radiative Recombination Coefficient

- The parameter $B$ is called the radiative recombination coefficient, and is typically quoted in units of cm$^3$s$^{-1}$.
- $B$ depends on the optical properties of the semiconductor of interest and the semiconductor.
- Typical values (at room temperature):

<table>
<thead>
<tr>
<th>Material</th>
<th>$B$ (cm$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>$4.73 \times 10^{-15}$</td>
</tr>
<tr>
<td>Ge</td>
<td>$5.2 \times 10^{-14}$</td>
</tr>
<tr>
<td>GaAs</td>
<td>$1.7 \times 10^{-10}$</td>
</tr>
<tr>
<td>GaP</td>
<td>$5.4 \times 10^{-14}$</td>
</tr>
<tr>
<td>InP</td>
<td>$1.8 \times 10^{-11}$</td>
</tr>
<tr>
<td>InSb</td>
<td>$4.6 \times 10^{-11}$</td>
</tr>
<tr>
<td>InGaAsP</td>
<td>$4 \times 10^{-10}$</td>
</tr>
</tbody>
</table>

Radiative Recombination Coefficient
• The parameter $B$ is called the radiative recombination coefficient, and is typically quoted in units of cm$^3$s$^{-1}$.
• $B$ depends on the optical properties of the semiconductor of interest and the semiconductor.
• Typical values (at room temperature):

- Lower numbers = faster recombination
- Higher numbers = slower recombination

High and Low-Level Limits

- Low-Level Injection:
  $\Delta n \ll p_0$
  $\tau_{rad}^{(ll)} \approx \frac{1}{Bp_0}$

- Very High Level Injection:
  $\Delta n \gg p_0$
  $\tau_{rad}^{(hl)} \approx \frac{1}{B\Delta n}$
Photon Recycling

- Before we conclude it is worth considering what happens to the photon that is emitted.
- In some systems (e.g. GaAs and perovskites) the photon can be re-absorbed: **Photon Recycling**.

Auger Recombination
Auger Recombination

- In a band-to-band Auger recombination process, conservation of potential energy associated with the annihilation of an electron-hole pair is achieved by transferring this potential energy to a third carrier, making it more energetic.
- There are two types of band-to-band Auger process, which can be distinguished as \( eeh \) or \( ehh \).
**eeh-Auger Recombination**

- First, we consider the *eeh*-Auger recombination process.
- Since we have two conduction band electrons and one valence band hole participate in such a process, we expect the rate \( R_{eeh} \) of such an Auger process to be proportional to the corresponding nonequilibrium carrier densities:
  \[
  R_{eeh} = C_n p n^2
  \]
- Where \( C_n \) is the proportionality constant for *eeh*-Auger recombination.

- In equilibrium, the rate of *eeh*-Auger recombination and generation must be equal.
- Equivalently, the net rate of *eeh*-Auger recombination must be equal to zero in equilibrium.
- Recognizing this allows us to write the net rate of *eeh*-Auger recombination as:
  \[
  U_{eeh} = C_n (p n^2 - p_0 n_0^2)
  \]
**eeh-Auger Recombination**

\[ U_{eeh} = C_n (pn^2 - p_0 n_0^2) \]

- Substituting nonequilibrium carrier concentrations by their equivalent excess carrier expressions and assuming charge neutrality:
  \[
  n = n_0 + \Delta n \\
  p = p_0 + \Delta p \\
  \Delta n = \Delta p
  \]

- Leads to:
  \[
  U_{eeh} = C_n (2p_0 n_0 + p_0 \Delta n + n_0^2 + 2n_0 \Delta n + \Delta n^2) \Delta n
  \]

---

**ehh-Auger Recombination**

- We can show, via a similar, analysis that the net rate for electron-hole-hole Auger recombination:
  \[
  U_{ehh} = C_p (2p_0 n_0 + n_0 \Delta n + p_0^2 + 2p_0 \Delta n + \Delta n^2) \Delta n
  \]

- Where \( C_p \) is the proportionality constant for ehh-Auger recombination.
Auger Lifetime

- Finally, the Auger lifetime can be given as:

\[ \tau_{\text{Auger}} = \frac{\Delta n}{U_{eeh} + U_{ehh}} \]

- Low-Level Injection:

\[ \Delta n \ll p_0 \]

\[ \tau^{(ll)}_{\text{Auger}} \approx \frac{1}{C_p p_0^2} \]

- Very High Level Injection:

\[ \Delta n \gg p_0 \]

\[ \tau^{(hl)}_{\text{Auger}} \approx \frac{1}{(C_n + C_p)\Delta n^2} \]

Lifetime Revisited
**Lifetime**

- Recall that we began this lecture with the recombination lifetime, $\tau_r$, which is the average time that a nonequilibrium minority carrier exists in a sea of majority carriers before it recombines.

\[
\frac{1}{\tau_r} = \frac{1}{\tau_{SRH}} + \frac{1}{\tau_{rad}} + \frac{1}{\tau_{Auger}}
\]

- Most of this lecture has been devoted to obtaining expressions for: $\tau_{SRH}$, $\tau_{rad}$, $\tau_{Auger}$.

- Now that we have these expressions, we are able to assess the lifetime, usually the minority carrier lifetime, using these expressions.

- As a last thought, recognize that there are two types of semiconductor device types, **bipolar** and **unipolar**.

- Bipolar devices include pn junctions, bipolar junction transistors and thyristors.

- Minority carrier lifetime is extremely important for bipolar devices.

- In fact, bipolar devices may be categorized as devices whose basic operation is determined exclusively by minority carrier action.

- Unipolar devices include Schottky barriers and FETs.
**Lifetime**

- Minority carrier lifetime is not relevant for unipolar devices.
- However, majority carrier lifetime may be important for certain classes of unipolar devices, in which the semiconductor is lightly doped.

**Next Time**

- Surface Recombination.