Question 1 [9 marks]:
a) Beyond use as intentionally-designed capacitor components, why is an extensive knowledge of capacitors valuable to the microelectronics industry?[2 marks]

Metal-oxide-semiconductor field-effect transistors (MOSFETs) are fundamental to integrated circuits and the microelectronics industry generally. The operation of MOSFETs fundamentally relies on charge being induced by capacitance, as in MOS capacitors. Hence much of the theory overlaps.

Similarly, integrated circuits will possess conducting components separated by insulating components. These structures will give rise to capacitive effects whether we want them or not.

b) If we operate an n-type MOS capacitor in accumulation mode, what type of charge carriers would you expect to observe at the interface between the semiconductor and insulator?[1 mark]

Electrons.

c) If we operate an n-type MOS capacitor in depletion mode, what type of charge carriers would you expect to observe at the interface between the semiconductor and insulator?[1 mark]

None / ionized impurities / lower concentration of electrons than usual.

d) If we operate an n-type MOS capacitor in inversion mode, what type of charge carriers would you expect to observe at the interface between the semiconductor and insulator? [1 mark]

Holes.

e) Figure 1 shows the capacitance per unit area of a MOS capacitor measured as a function of applied voltage (V) using a low frequency probe and a high frequency probe. From this data determine the capacitance the depletion region of this capacitor. Give your answer in μF/cm².[2 marks]

![Figure 1](image.png)

Figure 1 Capacitance per unit area of a MOS capacitor measured as a function of applied voltage (V) using a low frequency probe and a high frequency probe.
So we have two branches depending on the frequency at which we measure the capacitance. From Lecture 8 we know the in the low frequency regime (or in accumulation), the measured capacitance $C_{LF}$ is just the capacitance of the oxide (insulator): $C_i$:

$$C_{LF} = C_i$$

In the high frequency regime (or in depletion) the measured capacitance $C_{HF}$ is due to the capacitance of the oxide ($C_i$) in series with the capacitance of the depletion region ($C_D$). To add the capacitance of two capacitors in series we say:

$$\frac{1}{C_{HF}} = \frac{1}{C_D} + \frac{1}{C_i}$$

Re-arranging:

$$\frac{1}{C_D} = \frac{1}{C_{HF}} - \frac{1}{C_i}$$

$$\frac{1}{C_D} = \frac{C_i}{C_{HF}C_i} - \frac{C_{HF}}{C_{HF}C_i}$$

$$\frac{1}{C_D} = \frac{C_i - C_{HF}}{C_{HF}C_i}$$

$$C_D = \frac{C_{HF}C_i}{C_i - C_{HF}}$$

We know from above that $C_{LF} = C_i$:

$$C_D = \frac{C_{HF}C_{LF}}{C_{LF} - C_{HF}}$$

We can read the numbers off the graph (work in $\mu$F/cm$^2$):

$$C_D = \frac{2 \times 4}{4 - 2} = \frac{8}{2} = 4 \mu\text{F/cm}^2$$

f) Say we are carrying out a capacitance measurement on a silicon-based MOS capacitor. For this capacitor, the thermal generation time is known to be $1$ $\mu$s, the depletion width can be approximated to be $500$ nm, and the capacitance per unit area of the oxide is $20$ nF/cm$^2$. The intrinsic carrier concentration of silicon is $10^{10}$ cm$^{-3}$. Determine the voltage ramp-rate required to observe inversion in this semiconductor. Give your answer in V/s. [2 marks]

In Lecture 8 we were given the inequality describing the rate of change of voltage required to observe inversion:

$$\frac{dV}{dt} \leq \frac{en_iWA}{\tau_gC_i}$$
We are given all the parameters here. We are given the capacitance as capacitance per unit area. Make sure to convert to SI units:

\[
\frac{C_i}{A} = 20 \text{ nF/cm}^2 = 20 \times 10^{-9} \text{ F/cm}^2 = 2 \times 10^{-4} \text{ F/m}^2
\]

Putting everything in SI units:

\[
\frac{dV}{dt} \leq \frac{1.6 \times 10^{-19} \times 10^{16} \times 500 \times 10^{-9}}{1 \times 10^{-6} \times 2 \times 10^{-4}} = 4 \text{ V/s}
\]

**Question 2 [8 marks]:**

a) We are exposing a wafer to 365nm light through a 200 nm aperture, at a distance of 10µm from the wafer surface. What approximation for diffraction is most appropriate in this case: Fresnel or Fraunhofer?[2 marks]

To answer this question we need to evaluate the Fresnel number (F):

\[
F = \frac{W^2}{L\lambda}
\]

Where:
- \(W\) is the size of the aperture.
- \(L\) is the distance of the wafer from the aperture.
- \(\lambda\) is the wavelength of the incident light.

For \(F \ll 1\) the diffraction is described by Fraunhofer diffraction. For \(F \gg 1\) the diffraction is described by Fresnel diffraction.

Evaluating \(F\) (in nm):

\[
F = \frac{200^2}{10^4 \times 365} = 0.011
\]

Hence this situation is best described by Fraunhofer diffraction.

b) A photoresist that is deposited via spin-coating at a speed of 1,000 rpm has a thickness of 400nm. What would its thickness be if the speed was 4,000 rpm. [2 marks]

This is a simple problem, and can be solved intuitively. Here is a long answer for completeness.

We know that the film thickness \((t)\) is related to the angular velocity via this proportionality constant:

\[
t \propto \frac{1}{\sqrt{\omega}}
\]
We can change this into a proportionality relationship by introducing an constant $A$:

$$ t = \frac{A}{\sqrt{\omega}} $$

$$ t\sqrt{\omega} = A $$

We are told for a certain speed ($\omega_1=1,000$ rpm) we obtain a certain thickness ($t_1=400$ nm). Hence we can say that the proportionality constant is:

$$ A = 400\sqrt{1000} $$

Then to determine the thickness ($t_2$) for a different speed ($\omega_2 = 4,000$ rpm) we can use this constant:

$$ t_2 = \frac{A}{\sqrt{\omega_2}} $$

$$ t_2 = \frac{400\sqrt{1000}}{\sqrt{4000}} $$

$$ t_2 = 200$ nm

c) A positive photoresist with a contrast of $\gamma = 3$ is exposed to illumination, then developed to remove it from the wafer surface. If the incident optical power density is 200 mW/cm$^2$, this photoresist will remain unaltered by the development process up until an exposure of 120 seconds. How long must this photoresist be exposed to ensure that all the resist is removed during development?[4 marks]

To solve this problem we use the equation for contrast:

$$ \gamma = \frac{1}{\log_{10}\left(\frac{Q_f}{Q_0}\right)} $$

We are given the contrast: $\gamma = 3$, and we need to determine the amount of time we need to expose the photoresist for, to ensure that all of the resist is removed during development.

$Q_0$ here is the dose of optical radiation required to begin the exposure effect. What this means is that if we apply any dose $< Q_0$, then develop the film, 100% of the photoresist will remain. It is not until we reach this critical dose $Q_0$, that anything happens to the film upon development. From the text in this question we are told that “this photoresist will remain unaltered by the development process up until an exposure of 120 seconds” and we are also told that the optical power density is 200 mW/cm$^2$. Therefore we can determine the onset dose for exposure as:

$$ Q_0 = 200 \text{ mW/cm}^2 \times 120 \text{ s} = 24,000 \text{ mJ/cm}^2 $$

The parameter $Q_f$ is the dose at which exposure is complete. What this means is, that upon applying any dose of radiation of $Q_f$ or above, then developing the photoresist, 100% of the
photoresist will be removed. From the question, we seek the amount of time required for all photoresist to be removed upon development, i.e. we seek the time that gives rise to the dose $Q_f$.

So, re-arranging the equation for contrast in terms of $Q_f$:

$$\gamma = \frac{1}{\log_{10}\left(\frac{Q_f}{Q_0}\right)}$$

$$\log_{10}\left(\frac{Q_f}{Q_0}\right) = \frac{1}{\gamma}$$

$$\frac{Q_f}{Q_0} = 10^{1/\gamma}$$

$$Q_f = Q_010^{1/\gamma}$$

Putting in the numbers gives:

$$Q_f = 24,000 \times 10^{1/3}$$

$$Q_f = 5.17 \times 10^4 \text{ mJ/cm}^2$$

Now we just need to determine the amount of time required to provide this dose, from the incident optical power density (200 mW/cm$^2$).

$$t = \frac{Q_f}{I} = \frac{5.17 \times 10^4}{200} = 258 \text{ s}$$

**Question 3 [4 marks]:**

The table below shows some published values of the growth parameters of SiO$_2$ on silicon. For the dry growth process, the parameter $\tau$ is already evaluated.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>A (μm)</th>
<th>B (μm²/hour)</th>
<th>$\tau$ (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>0.370</td>
<td>0.0011</td>
<td>9</td>
</tr>
<tr>
<td>920</td>
<td>0.235</td>
<td>0.0049</td>
<td>1.4</td>
</tr>
<tr>
<td>1000</td>
<td>0.165</td>
<td>0.0117</td>
<td>0.37</td>
</tr>
<tr>
<td>1100</td>
<td>0.090</td>
<td>0.027</td>
<td>0.076</td>
</tr>
<tr>
<td>1200</td>
<td>0.040</td>
<td>0.045</td>
<td>0.027</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wet (640 torr)</th>
<th>A (μm)</th>
<th>B (μm²/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>0.50</td>
<td>0.203</td>
</tr>
<tr>
<td>920</td>
<td>0.226</td>
<td>0.287</td>
</tr>
<tr>
<td>1000</td>
<td>0.11</td>
<td>0.510</td>
</tr>
<tr>
<td>1100</td>
<td>0.08</td>
<td>0.720</td>
</tr>
<tr>
<td>1200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) At a temperature of 1100°C, what starting oxide thickness does this data for the dry oxide growth assume? [2 marks]

This is from Lecture 12.

Here we are given $\tau$ for dry oxide growth, as well as $A$ and $B$. So we can just use our equation for $\tau$:
\[
\tau = \frac{(x_0^2 + A x_0)}{B}
\]

We need to solve this for \(x_0\):

\[
B \tau = x_0^2 + A x_0
\]

Writing this in standard quadratic form:

\[
x_0^2 + A x_0 - B \tau = 0
\]

We can then use the quadratic equation:

\[
x_0 = \frac{-A \pm \sqrt{A^2 + 4 B \tau}}{2}
\]

Now we just enter values from the table:

\[
x_0 = \frac{-0.090 \pm \sqrt{0.090^2 + 4 \times 0.027 \times 0.076}}{2}
\]

(Alternatively, you could have just used the equation from the lectures and said \(x_0 = x_{ox}\) when \(t=0\)).

For the dry oxide growth at 1100°C, we can see these values are:

\(A = 0.090 \ \mu\text{m}\).

\(B = 0.027 \ \mu\text{m}^2/\text{hr}\).

\(\tau=0.076 \ \text{hr}\).

\[
x_0 = \frac{-0.090 \pm \sqrt{0.0163}}{2}
\]

\[
x_0 = \frac{-0.090 \pm 0.128}{2}
\]

Since \((-0.09-0.128)\) is negative, only the positive root is physically meaningful:

\[
x_0 = \frac{-0.090 + 0.128}{2}
\]

\[
x_0 = \frac{0.0377}{2}
\]

\[
x_0 = 0.0189 \ \mu\text{m}
\]

b) Using the parameters in the above table, determine total thickness if a dry oxidation was carried out for 5 hours at 1100°C. [2 marks]

This is a simple exercise of putting the numbers into the equation:

\[
x_{ox} = \frac{-A \pm \sqrt{A^2 + 4 B (t + \tau)}}{2}
\]
\[ x_{ox} = -0.09 + \sqrt{0.09^2 + 4 \times 0.027 \times (5 + 0.076)} \]

Giving:

\[ x_{ox} = 0.330 \text{μm} \]

Question 3 [4 marks]:

The table below shows the values of \( D_0 \) and the activation energy \( E_A \) for diffusion of some species in Si.

<table>
<thead>
<tr>
<th>( E_V ) (eV)</th>
<th>( E_D ) (cm(^2)/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (substitutional)</td>
<td>3.85</td>
</tr>
<tr>
<td>B (substitutional)</td>
<td>0.037</td>
</tr>
<tr>
<td>As (substitutional)</td>
<td>0.066</td>
</tr>
<tr>
<td>Sb (substitutional)</td>
<td>0.214</td>
</tr>
<tr>
<td>Au (interstitial)</td>
<td>0.0011</td>
</tr>
<tr>
<td>Cu (interstitial)</td>
<td>0.04</td>
</tr>
<tr>
<td>Fe (interstitial)</td>
<td>0.0062</td>
</tr>
<tr>
<td>O (interstitial)</td>
<td>0.21</td>
</tr>
</tbody>
</table>

a) To n-dope a silicon wafer we deposit boron atoms on top of the silicon, then anneal to diffuse the dopant atoms into the wafer. If we anneal our wafer at 1300K for 1 hour, what will the concentration of boron be at a distance of 50nm from the surface?[4 marks]

Assume that on top of the wafer is a limitless source of boron, with concentration \( 10^{18} \text{ cm}^{-3} \), and we can hence use the following equation to describe the concentration as a function of position and time:

\[ C(z, t) = C_0 \text{erfc} \left( \frac{z}{2\sqrt{D_0 t}} \right) \]

We can approximate the complementary error function as:

\[ \text{erfc}(x) = (a_1 y + a_2 y^2 + a_3 y^3) e^{-x^2} \]

Where:

\[ y = \frac{1}{1 + px} \]

And:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>0.3480242</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>-0.0958798</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>0.7478556</td>
</tr>
<tr>
<td>( p )</td>
<td>0.47047</td>
</tr>
</tbody>
</table>
This a case of entering values into equations. Determine the argument for the complementary error function:

\[ x = \frac{z}{2\sqrt{Dt}} \]

We need to first evaluate the diffusion coefficient for boron:

\[ D = D_0 \exp \left( -\frac{E_{VD}}{k_BT} \right) \]

\[ D = 0.037 \times \exp \left( -\frac{3.46 \times 1.602 \times 10^{-19}}{1.38 \times 10^{-23} \times 1300} \right) \]

\[ D = 1.41 \times 10^{-15} \text{ cm}^2\text{s}^{-1} \]

Evaluate distance from the surface of the wafer in cm:

\[ z = 50 \text{ nm} = 5 \times 10^{-6} \text{ cm} \]

Enter values:

\[ x = \frac{5 \times 10^{-6}}{2\sqrt{1.41 \times 10^{-15} \times 3600}} = 1.109 \]

Now we just use the approximation for the complementary error function.

Evaluate each term:

\[ y = \frac{1}{1 + 0.47047 \times 1.109} = 0.657 \]

\[ a_1 y = 0.229 \]
\[ a_2 y^2 = -0.0414 \]
\[ a_3 y^3 = 0.212 \]
\[ e^{-x^2} = 0.292 \]

\[ \text{erfc}(x) = (0.229 - 0.0414 + 0.212) \times 0.292 = 0.117 \]

So we can then finally evaluate the concentration:

\[ C(z, t) = C_0 \text{erfc}\left( \frac{z}{2\sqrt{Dt}} \right) \]

\[ C(z, t) = 10^{18} \times 0.117 = 1.17 \times 10^{17} \text{ cm}^{-3} \]