ECE 627

Spring 2014

Final Examination

June 13, 2013, 9:30 – 11:20 am

1. A ΔΣ ADC has an *NTF* = $(1 - z^{-1})^3$. The quantizer output step size is Δ = 2. Assuming that the error samples *e(n)* are uncorrelated, find the power (mean-square value) of the quantization noise in the output signal.

Output noise samples $q(n)$ from

$$Q(z) = (1 - 3z^{-1} + 3z^{-2} - z^{-3})\, E(z):$$

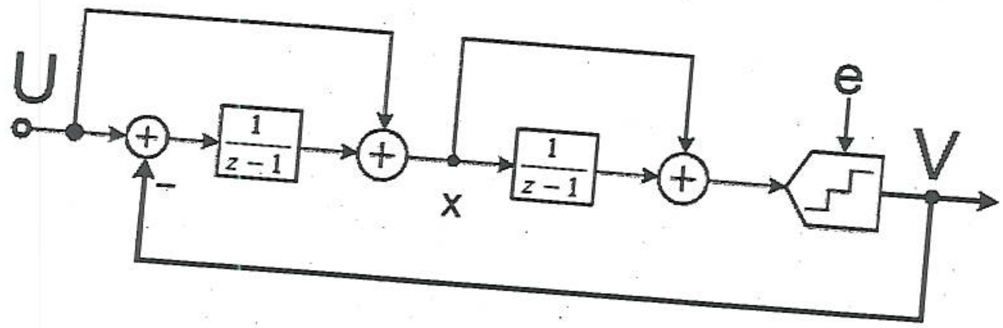
$$q(n) = e(n) - 3e(n-1) + 3e(n-2) - e(n-3)$$

If $e(k)$ are uncorrelated

$$\overline{q^2(n)} = 20\,\overline{e^2(n)} = 20\,\Delta^2/12 = 20/3 \cong 6.67$$

2. a. Find the NTF and STF for the $\Delta\Sigma$ loop shown.

b. What is the signal $V_x(z)$ at node X?



$$2.\ a.\quad V = E + (I+1)\left[U + (U-V)\, I\right]$$

$$V\left[1 + I(I+1)\right] = E + U(I+1)^2$$

$$I + 1 = \frac{z}{z-1} = zI$$

$$V\left(1 + zI^2\right) = E + z^2 I^2 U$$

$$NTF = 1/\left[1 + z/(z-1)^2\right] = \frac{(z-1)^2}{(z-1)^2 + z} = \frac{(1-z^{-1})^2}{z^{-2} - z^{-1} + 1}$$

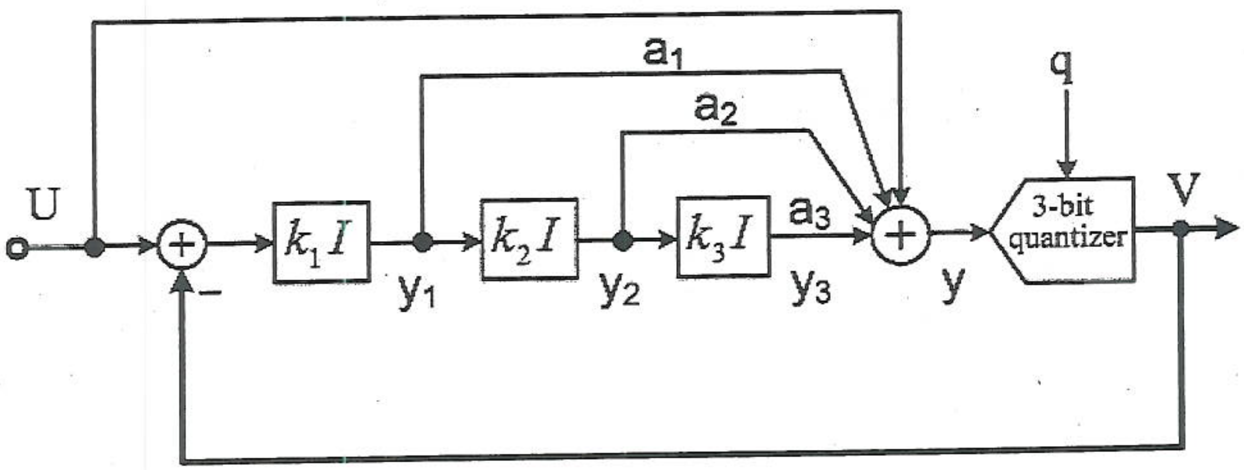
$$STF = z^2/\left[(z-1)^2 + z\right] = 1/(z^{-2} - z^{-1} + 1)$$

$$b.\quad V_x = U + I(U - V) = \frac{(1-z^{-1})(U - z^{-1}E)}{z^{-2} - z^{-1} + 1}$$

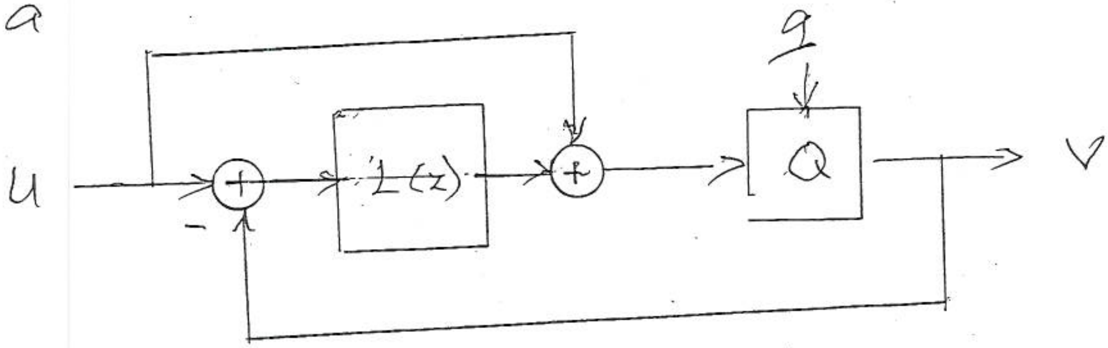
3. a. The $\Delta\Sigma$ ADC shown below has $NTF = (1 - z^{-1})^3$. The integrators are delaying. What is the STF?

b. Choose k_1, k_2 and k_3 such that for zero input (u = 0) the largest output swings of all integrators equal $V_{ref}/2$. Here, $V_{ref} = 8V_{LSB}$ is the full-scale reference voltage of the quantizer and the DAC.

c. Find a_1, a_2 and a_3 to obtain the NTF given above.



3.a



$$V = Q + u + (u - V)L$$

$$V(1+L) = Q + u(1+L)$$

$$STF = 1, \quad NTF = 1/(1+L)$$

3. b. $V = U + HQ$, $H = NTF = (1-z^{-1})^3$

$$Y_1 = k_1 I(U-V) = k_1 \frac{z^{-1}}{1-z^{-1}}(-HQ) = -k_1 z^{-1}(1-z^{-1})^2 Q$$

$$|y_1| = k_1 |q(n-2) - 2q(n-1) + q(n)| \le 4 k_1 V_{ref}/8 \overset{!!}{=} V_{ref}/2$$

$\underline{k_1 = 1}$

$$-Y_2 = k_1 k_2 I^2 HQ = k_2(1-z^{-1}) z^{-2} Q$$

$$|y_2| = |k_2 (q_{n-2} - q_{n-3})| \le 2 k_2 V_{ref}/8$$

$\underline{k_2 = 2}$

$$-Y_3 = k_1 k_2 k_3 I^3 HQ = 2 k_3 z^{-3} Q$$

$$|y_3| = 2 k_3 V_{ref}/8 \overset{!}{=} V_{ref}/2$$

$\underline{k_3 = 2}$

c. $Y = U + a_1 Y_1 + a_2 Y_2 + a_3 Y_3 = U + (H-1)Q$

$$H = 1 - a_1 z^{-1}(1-z^{-1})^2 - a_2 2 z^{-2}(1-z^{-1}) - a_3 4 z^{-3}$$

$$\overset{!}{=} 1 - 3z^{-1} + 3z^{-2} - z^{-3}$$

$z = 1 \rightarrow a_3 = 1/4$, z^{-1} coeff: $a_1 = 3$

z^{-2} coeff: $-2a_1 + 2a_2 = 3$, $a_2 = 3/2$

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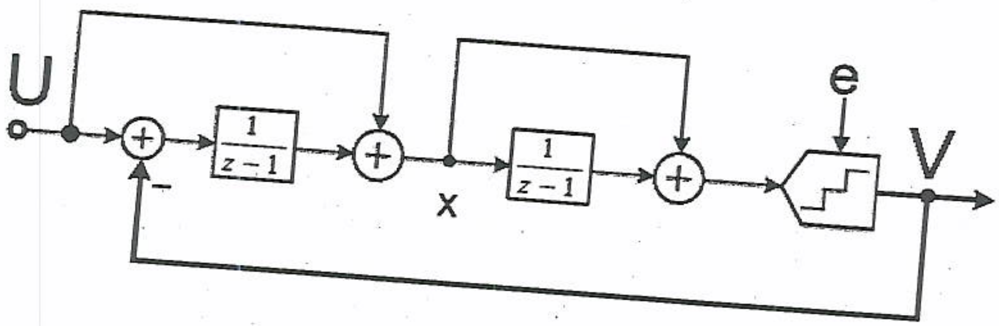

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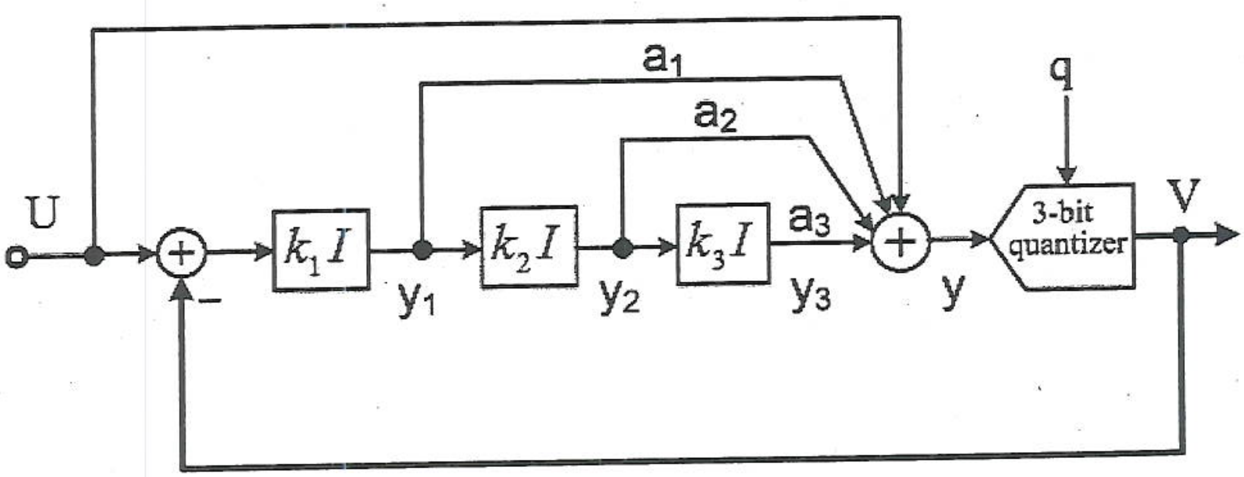

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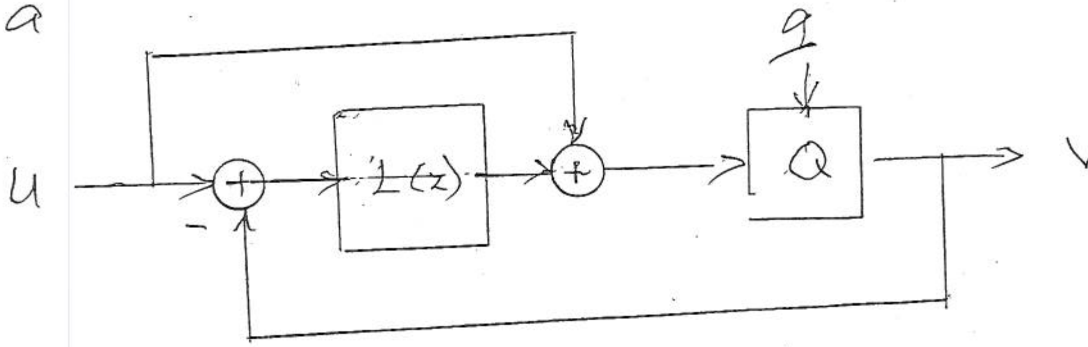

$V = Q + u + (u - V)L$

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$STF = 1$, $NTF = 1/(1 + L)$

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$$\underline{k_2 = 2}$$

$$-Y_3 = k_1 k_2 k_3 I^3 HQ = 2 k_3 z^{-3} Q$$

$$|y_3| = 2 k_3 V_{ref}/8 \stackrel{!}{=} V_{ref}/2$$

$$\underline{k_3 = 2}$$

c. $Y = U + a_1 Y_1 + a_2 Y_2 + a_3 Y_3 = U + (H-1)Q$

$$H = 1 - a_1 z^{-1}(1-z^{-1})^2 - a_2 2 z^{-2}(1-z^{-1}) - a_3 4 z^{-3}$$

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