ECE 627

Spring 2015

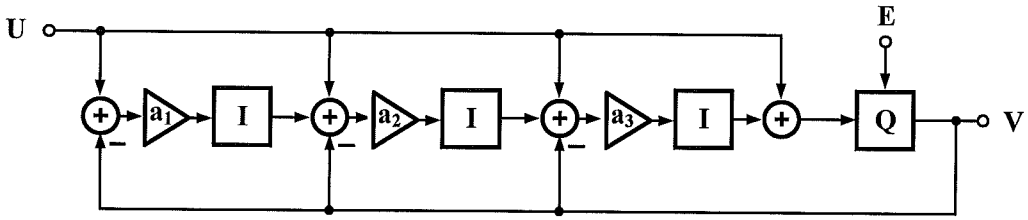
Final Examination

June 11, 2015, 6 - 7:50 pm

Open book

1.a. Find the transfer functions *STF*(z) and *NTF*(z) of the ΔΣ ADC shown below. Use $I(z) = \frac{1}{z-1}$.

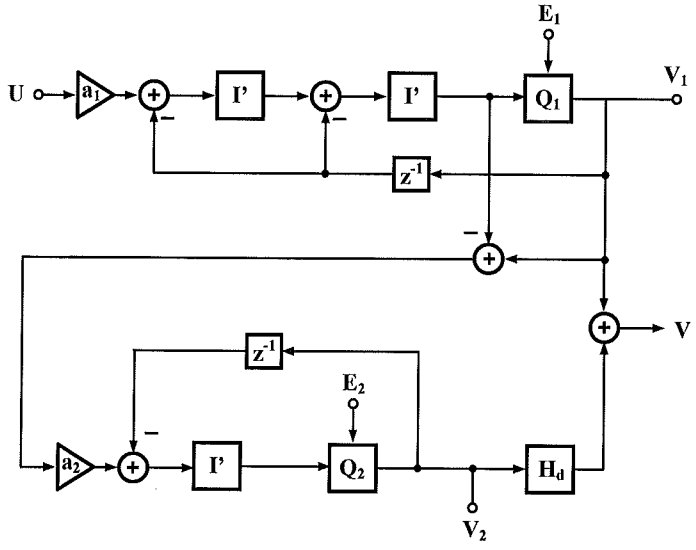
b. Find the values of a_1, a_2 and a_3 which give $NTF(z) = (1 - z^{-1})^3$.



2.a. Find the output signals V_1 and V_2 in the MASH ADC shown, as functions of U, E_1 and E_2. Use $I'(z) = \frac{z}{z-1}$.

b. What should $H_d(z)$ be chosen in order to cancel E_1 in the overall output $V(z)$?

c. What is the overall output signal $V(z)$ for this choice of $H_d(z)$?



1. $V = E + U + (U-V)\{a_3 I + a_2 a_3 I^2 + a_1 a_2 a_3 I^3\}$

$[1 + a_3 I + a_2 a_3 I^2 + a_1 a_2 a_3 I^3]V = [\ ]U + E$

$STF \equiv 1$, $NTF = 1/[\ ] = (z-1)^3 / [(\ )^3 + a_3(\ )^2 + a_2 a_3(\ ) + a_1 a_2 a_3]$

$I = 1/(z-1)$

Let $a_3 = 3$, $a_2 = 1$, $a_1 = 1/3$

$NTF = (z-1)^3 / [(z-1)+1]^3 = (1-z^{-1})^3$

$z = 1 + 1/I = \frac{I+1}{I}$

$1 - z^{-1} = 1 - \frac{I}{I+1} = \frac{1}{I+1}$

$(1-z^{-1})^3 = 1/(I+1)^3$

$I^3 + 3I^2 + 3I + 1 \overset{!}{=} a_1 a_2 a_3 I^3 + a_2 a_3 I^2 + a_3 I +$

$a_3 = 3$, $a_2 = 1$, $a_1 = 1/3$

2. $V_1 = E_1 + I'\left[-z^{-1}V_1 + I'\left[a_1 U - z^{-1}V_1\right]\right]$

$= E_1 + a_1 I'^2 U - z^{-1}V_1\left(I' + I'^2\right)$

$\left[1 + z^{-1}I' + z^{-1}I'^2\right]V_1 = E_1 + a_1 I'^2 U$

$NTF_1 = 1/[\ ] = 1/\left[1 + \frac{1}{z-1} + \frac{z}{(z-1)^2}\right]$

$= (z-1)^2 / \left[z^2 - 2z + 1 + z - 1 + z\right] = (1-z^{-1})^2$ ✓

$STF_1 = a_1 \frac{z^2}{z^2} = a_1$ ✓

$V_2 = E_2 + I'\left[a_2 E_1 - z^{-1}V_2\right]$

$(1 + z^{-1}I')V_2 = E_2 + a_2 I' E_1$, $\left|\begin{array}{l} 1 + z^{-1}I' = 1 + \frac{1}{z-1} \\ -\frac{z}{z-1} = \frac{1}{1-z^{-1}} \end{array}\right.$

$V_2 = (1-z^{-1})E_2 + a_2 E_1$ ✓

$NTF_1 E_1 + H_d a_2 E_1 \doteq 0$

$H_d \doteq -NTF_1 / a_2 = -(1-z^{-1})^2 / a_2$

$V = V_1 + H_d V_2 = a_1 U + (1-z^{-1})^2 E_1 -$
$(1-z^{-1})^3 E_2 - (1-z^{-1})^2 E_1$

$V = a_1 U - (1-z^{-1})^3 E_2 / a_2$