ECE 627

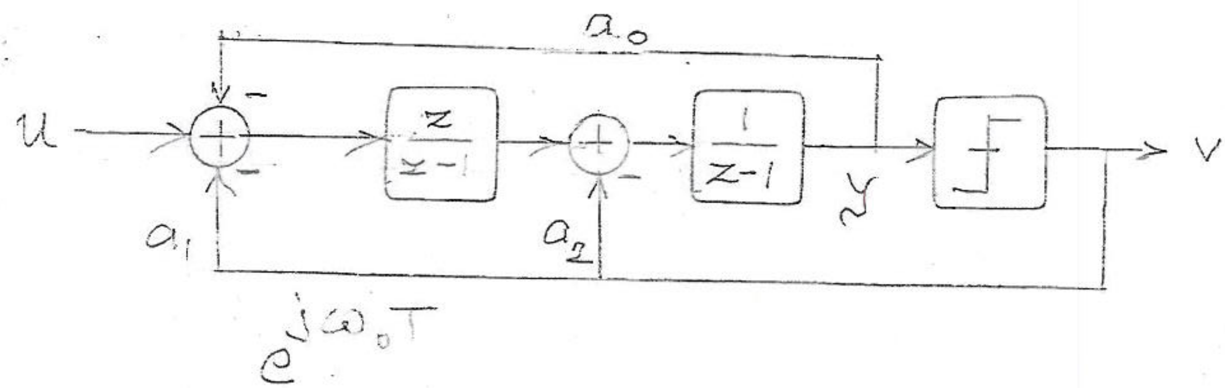
Spring 2013

Final Examination

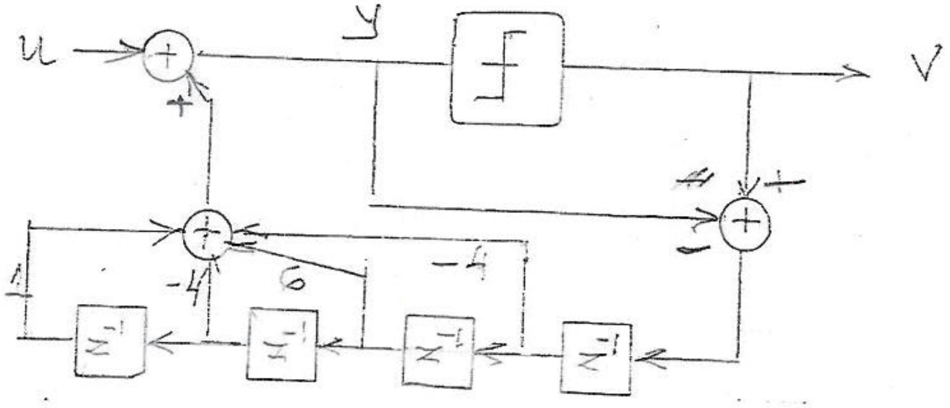
June 11, 2013, 2 – 3:50 pm

1. a. Find the constants a_i in the ΔΣ ADC shown below such that the $STF = z^{-1}$ and the $NTF = z^{-2} - 1.95z^{-1} + 1$.

b. Plot $|NTF|$ for $0 < f < f_s/2$.



2. Find the NTF and STF for the ΔΣ loop shown.



1.a $V = E + I_2\left\{-a_2V + I_1\left[U - a_1V - a_0V + a_0E\right]\right.$

$$V\left[1 + a_2I_2 + (a_0 + a_1)I_1I_2\right] = (1 + a_0I_1I_2)E + I_1I_2U$$

$$1/STF = \frac{1}{I_1I_2} + \frac{a_2}{I_1} + a_0 + a_1 = \frac{(z-1)^2}{z} + \frac{z-1}{z}a_2 + a_0 + a_1$$

$$z = z - 2 + z^{-1} + a_2(1 - z^{-1}) + a_0 + a_1$$

$$a_2 = 1, \quad a_0 + a_1 + a_2 = 2$$

$$NTF = \frac{1 + a_0I_1I_2}{1 + a_2I_2 + (a_0 + a_1)I_1I_2} = \frac{(z-1)^2 + a_0z}{\text{denom}}$$

$$\text{denom} = (z-1)^2 + (z-1) + (a_0 + a_1)z \doteq z^2$$

$$-2z + 1 + z - 1 + (a_0 + a_1)z = 0$$

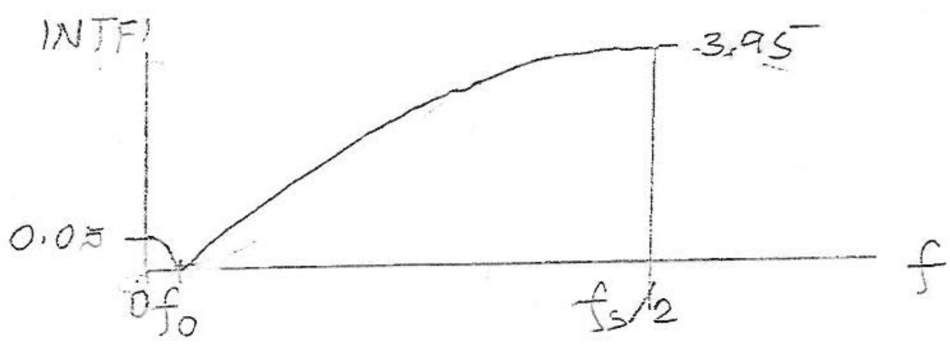
$$a_0 + a_1 = 1, \quad a_0 = 0.05, \quad a_1 = 0.95$$

1.b From diagram, NTF zero is at $e^{\pm j\omega_0T}$

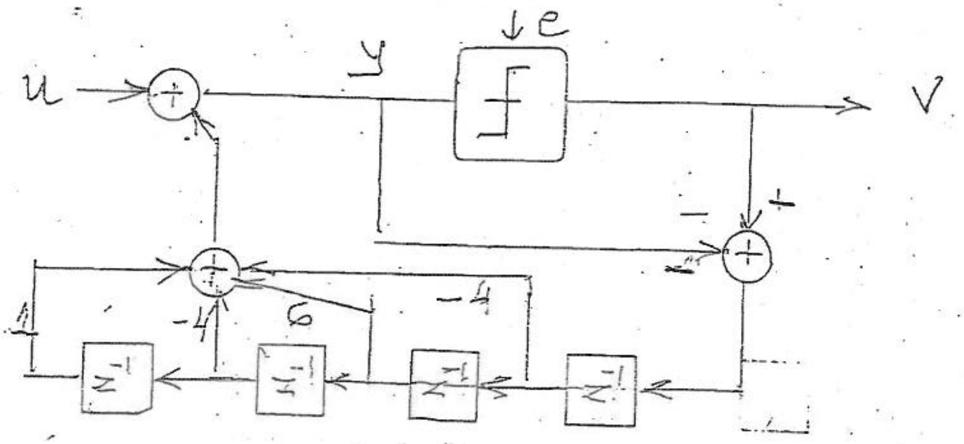
$$z^2 - 1.95z + 1 = (z - e^{j\omega_0T})(z - e^{-j\omega_0T})$$

$$\cos\omega_0T = 1.95/2 \quad \text{or} \quad 1 - \frac{(\omega_0T)^2}{2} \cong 0.975$$

$$f_0 = \frac{f_s}{2\pi}\cos^{-1}0.975 \cong 0.0356\,f_s$$



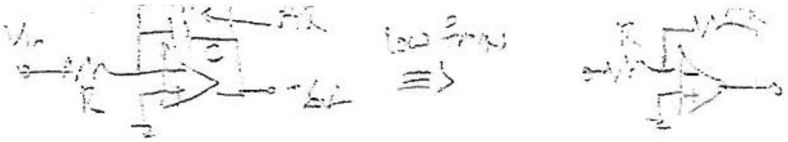
2. Find the NTF and STF for the $\Delta\Sigma$ loop shown.



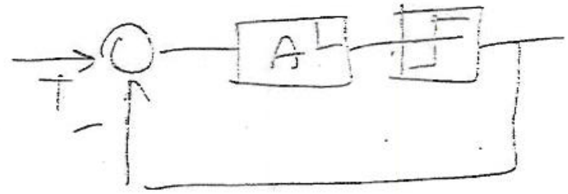
$$V = E + U + E\left(-4z^{-1} + 6z^{-2} - 4z^{-3} + z^{-4}\right) = U + (1-z^{-1})^4 E$$

$$STF = 1, \quad NTF = (1-z^{-1})^4$$

each integrator :



3.a) At very low freqs the loop Gain is A^L



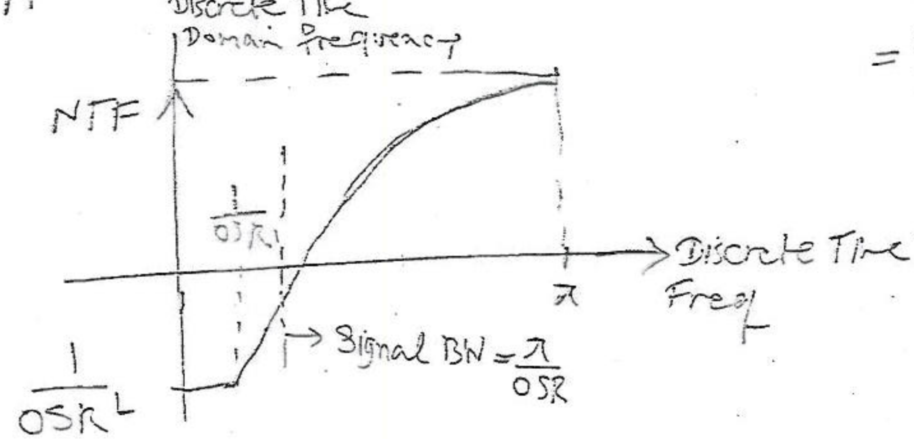
so the $NTF = \dfrac{1}{1+A^L} \simeq \dfrac{1}{A^L}$

$$|NTF| = \left|(1-z^{-1})^L\right|_{z=e^{-j\omega}} = \left|(1-e^{-j\omega})^L\right| \simeq \omega^L$$

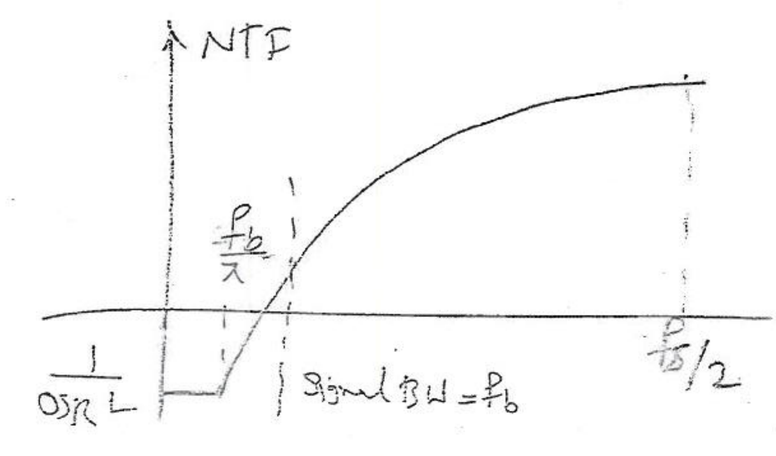
at low freqs

$$\omega^L = \frac{1}{A^L} \Rightarrow \omega = \frac{1}{A} = \frac{1}{OSR} \Rightarrow \text{Continuous Time freq} = \frac{1}{OSR} \times \frac{f_s}{2\pi}$$

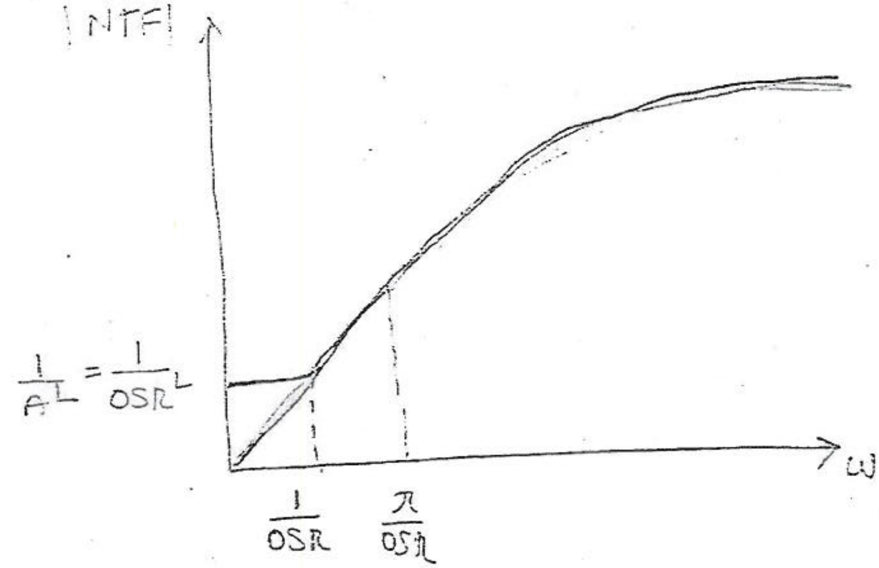
$$= \frac{2f_b}{f_s} \times \frac{f_s}{2\pi} = \frac{f_b}{\pi}$$



The



3.b



Noise power: $\frac{\Delta^2}{12\pi}\left(\int_0^{\frac{1}{OSR}} \frac{1}{OSR^{2L}} d\omega - \int_{\frac{1}{OSR}}^{\frac{\pi}{OSR}} \omega^{2L} d\omega \right)$

$\left(\begin{array}{l} \text{Noise transfer function} \\ \text{in } 0 < \omega < \frac{1}{OSR} \end{array} \right)^2$

$\left(\begin{array}{l} \text{Noise transfer function} \\ \text{in } \frac{1}{OSR} < \omega < \frac{\pi}{OSR} \end{array} \right)^2$

$$= \frac{\Delta^2}{12\pi}\left(\frac{1}{OSR^{2L+1}} + \frac{1}{2L+1}\left(\frac{\pi^{2L+1}}{OSR^{2L+1}} - \frac{1}{OSR^{2L+1}} \right) \right)$$

$$= \frac{\Delta^2}{12\pi}\left(\frac{1}{OSR^{2L+1}} \frac{2L}{2L+1} + \frac{1}{2L+1} \frac{\pi^{2L+1}}{OSR^{2L+1}} \right)$$

Ideal Noise Power $= \frac{\Delta^2}{12\pi} \int_0^{\frac{\pi}{OSR}} \omega^{2L} d\omega = \frac{\Delta^2}{12\pi} \frac{1}{2L+1} \frac{\pi^{2L+1}}{OSR^{2L+1}}$

$$\frac{\text{New Noise Power}}{\text{Ideal Noise Power}} = \frac{\frac{\Delta^2}{12\pi}\left(\frac{1}{OSR^{2L+1}} \frac{2L}{2L+1} + \frac{1}{2L+1} \frac{\pi^{2L+1}}{OSR^{2L+1}} \right)}{\frac{\Delta^2}{12\pi} \frac{1}{2L+1} \frac{\pi^{2L+1}}{OSR^{2L+1}}}$$

$$\frac{\text{New Noise Power}}{\text{Ideal Noise Power}} = 1 + \frac{2L}{x^{2L+1}}$$

Amount of decrease in SQNR $= 10 \log \left(1 + \frac{2L}{x^{2L+1}} \right)$