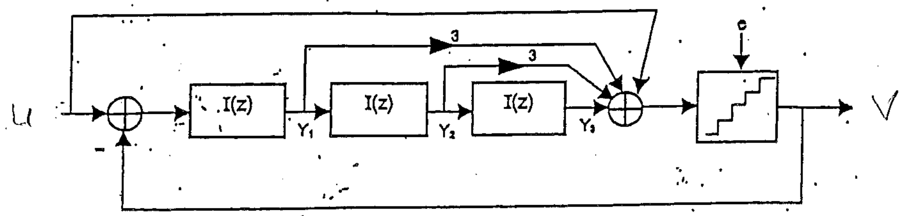
ECE 627

Spring 2016

Final Examination

June 6, 2015, 9:30 – 11:20 am

1. In the circuit shown, $I(z) = 1/(z - 1)$. Find the integrator outputs in the time domain. Assuming that the quantizer is not overloaded, so $|e| < V_{LSB}/2$ , what are the largest values of $y_1$, $y_2$ and $y_3$?

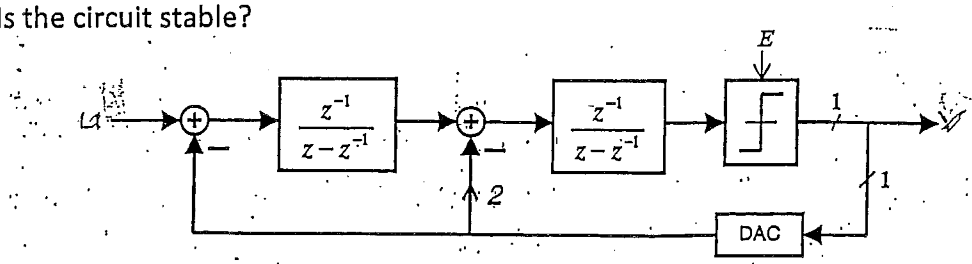



2 a. Find the transfer functions $STF(z)$ and $NTF(z)$ of the ΔΣ ADC shown. Use $I(z) = 1/(z^2-1)$. (Note the unusual functions!)

b. Where are the zeros and poles of $STF(z)$ and $NTF(z)$?

c. Draw $|NTF|$ as a function of frequency for $0 < f < f_s/2$.

d. Is the circuit stable?

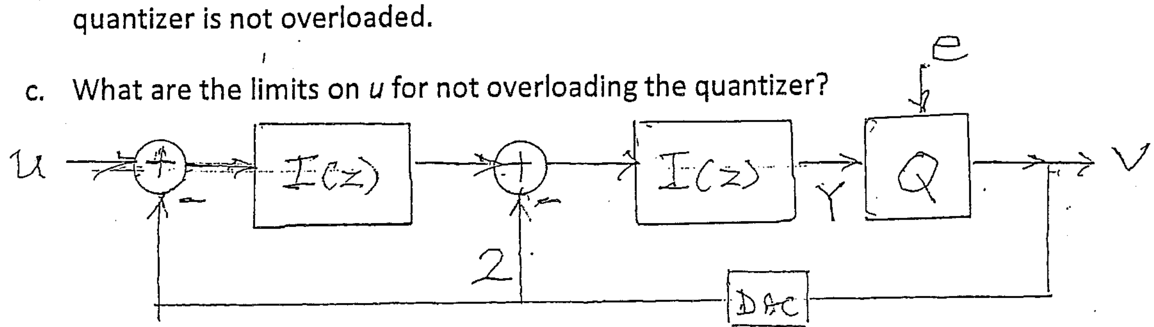



3. In the ΔΣ ADC shown, $I(z) = 1/(z - 1)$. Two-bit quantization is used, and $V_{ref} = 3$ V.

a. Find the *NTF* and *STF*.

b. Find the largest and smallest values of $y(n)$ for a given dc input $u$, assuming that the quantizer is not overloaded.

c. What are the limits on $u$ for not overloading the quantizer?



1. $V = E + U + (U - V)(3I + 3I^2 + I^3)$

$$V(1+I)^3 = U(1+I)^3 + E$$

$$STF = 1\ ,\quad NTF = (1+I)^{-3} = (1-z^{-1})^3$$

$$Y_1 = I(U-V) = -I(1-z^{-1})^3 E = -z^{-1}(1-z^{-1})^2 E$$

$$y_1 = -e(n-1) + 2e(n-2) - e(n-3)$$

$$|y_1| \le 4|e|_{max} = 2V_{LSB}$$

$$Y_2 = -I^2(1-z^{-1})^3 E = -z^{-2}(1-z^{-1}) E$$

$$y_2 = -e(n-2) + e(n-3)$$

$$|y_2| \le 2|e|_{max} = V_{LSB}$$

$$Y_3 = -I^3(1-z^{-3})^3 E = -z^{-3} E$$

$$y_3 = -e(n-3)$$

$$|y_3| \le |e|_{max} = V_{LSB}/2$$

$$2.a \quad V = E + I\left[-2V + I(U-V)\right]$$

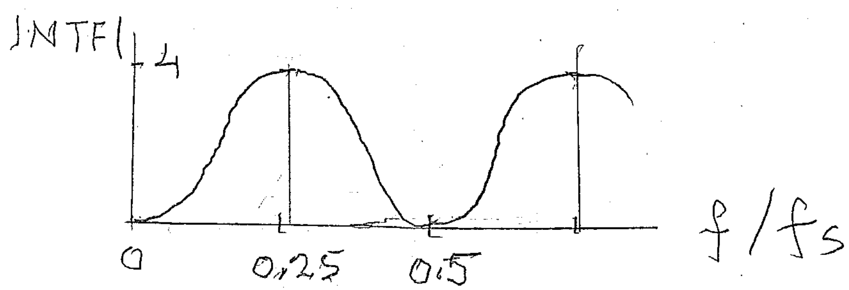
$$V(1+2I+I^2) = E + I^2 U$$

$$(1+I)^2 = \left[1 + \frac{1}{z^2-1}\right]^2 = \left[\frac{z^2}{z^2-1}\right]^2$$

$$NTF = \frac{(z^2-1)^2}{z^4}$$ zeros at $z = \pm 1$ (double), 4 poles at $z=0$

$STF = z^{-4}$; 4 poles at $z=0$

b.




c. The NTF is that of MOD2, with $z$ replaced by $z^2$, same for STF. So both frequency responses are compressed by 2, and the time responses strechted by 2. Since MOD2 is stable, so is this system, for a range of inputs.

3\. a. $V = E + I[-2V + I(U - V)]$

$$V(1 + 2I + I^2) = E + I^2 U$$

$$(I+1)^2 = \frac{z^2}{(z-1)^2}$$

$$NTF = (1 - z^{-1})^2$$

$$STF = z^{-2}$$

b. $Y = V - E = z^{-2}U + [(1-z^{-1})^2 - 1]E$

$$y(n) = u(n-2) - 2e(n-1) + e(n-2)$$

For the quantizer not overloaded, $|e| \leq V_{LSB}/2 = 3/8\ V$

Hence,

$$u - 9/8 \leq y \leq u + 9/8$$

c. To avoid overloading the quantizer, $-V_{LSB}/2 \leq y \leq V_{ref} + V_{LSB}/2$ must hold. Hence

$$u_{min} - 9/8 \geq -V_{LSB}/2 \rightarrow u_{min} \geq 0.75\ V$$

$$u_{max} + 9/8 \leq 3 - 3/8 \rightarrow u_{max} \leq 1.5\ V$$