CS 325
Due: Friday, 8 Feb

Homework #4

1. Solve, the following difference equation initial value problems. If the problem is nonnegative, find the "Big Oh" order of the solution and compare it to the "Big Oh" order predicted in the Notes (Ch. 9). (Suggestion: Use one of the mathematical computation packages to solve these DE's and compare the package's solution to your solution.)

   (a) $x_n = 2x_{n-1} \qquad x_0 = 1$
   (b) $x_n = 2x_{n-1} - 1 \qquad x_0 = 0$
   (c) $x_n = 2x_{n-1} - 1 \qquad x_0 = 1$
   (d) $x_n = 2x_{n-1} + n + 17 \qquad x_0 = 1$
   (e) $x_n = 2x_{n-1} + 2^n \qquad x_0 = 7$
   (f) $x_n = 2x_{n-1} + 3^n \qquad x_0 = 7$
   (g) $f_n = f_{n-1} + f_{n-2} \qquad f_1 = 2 \quad f_2 = 1$
   (h) $f_n = f_{n-1} + f_{n-2} + 1 \qquad f_1 = f_2 = 0$
   (i) $f_n = f_{n-1} + f_{n-2} - n \qquad f_0 = 3 \;\; f_1 = 4$
   (j) $x_n = 2x_{n-1} + n \qquad x_0 = 0$
   (k) $x_n = 2x_{n-1} + n \qquad x_0 = -1$
   (l) $x_n = 2x_{n-1} - n \qquad x_0 = 0$
   (m) $x_n = 3x_{n-1} + 3^n \qquad x_1 = 3$
   (n) $x_n = 3x_{n-1} + 3^n \qquad x_0 = 3$
   (o) $f_{n+1} = f_n + f_{n-1} + 4n - n^2 \qquad f_1 = 1 \quad f_2 = 4$
   (p) $x_n = 2x_{n-1} + n^2 \qquad x_1 = 2$
   (q) $x_n = x_{n-1} + 2x_{n-2} \qquad x_0 = 0, \; x_1 = 1$
   (r) $x_n = x_{n-1} + 2x_{n-2} + 3^n \qquad x_0 = 0, \; x_1 = 1$

2. An AVL tree is a binary tree in which at each vertex the heights of the right and left subtrees of that vertex differ by at most 1. If the AVL tree is of height h what is the maximum possible number of vertices in the tree? What is the minimum number of vertices? Derive difference equations for the maximum and the minimum number of vertices in an AVL tree. Find appropriate initial conditions for these difference equations. Solve both these initial value problems and thus obtain bounds on the number of vertices in an AVL tree of height h.

3. For the following algorithms given in schematic form, write down the difference equation for the run time.

(a)

```
FUNCTION   FOX( X, n)
               Split X into  X1 and X2
               Y1 = FOX(X1, n/2)
               Y2 = FOX(X2, n/2)
               COMBINE(Y1, Y2)
```

If SPLIT and COMBINE are both $\Theta(n)$ then FOX has runtime

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(b)

```
PROCEDURE   MESS( X, n)
               Split X into  Y1,  Y2,  Y3
               MESS( Y1, n/3)
               MESS( Y2, n/3)
               MESS( Y3, n/3)
               COMBINE the results of the subproblems
```

If SPLIT and COMBINE are $\Theta(n^2)$ then MESS has runtime

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(c) If SPLIT and COMBINE are both $\Theta(1)$ then MESS from (b) has runtime

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(d)

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PROCEDURE   MULT( a, b, n)
               Split a into  a0, a1, and b into b0, b1
               MULT( a0, b0, n/2)
               MULT( a1, b1, n/2)
               MULT( a1 + a0, b1 + b0, n/2)
               COMBINE the results of the subproblems
```

If COMBINE has $\Theta(n)$ run time then MULT has runtime

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