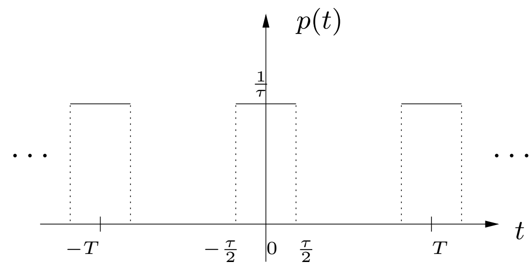
ECE352: Signals & Systems II
Dr. Raviv Raich
Midterm I (Feb. 11, 2008)

The duration of the exam is fifty minutes (3:00pm-3:50pm). Only one sheet of formulae is allowed. No calculators are allowed. Return this copy of the exam form along with formulae sheet and your notes.

1. (50%) Let $p(t)$ be the periodic signal with period T defined in $[-T/2, T/2]$ by:

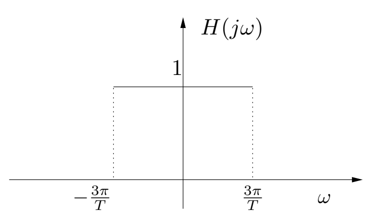
$$p(t) = \begin{cases} \frac{1}{\tau} & |t| < \frac{\tau}{2} \\ 0 & |t| \geq \frac{\tau}{2} \end{cases}$$



(a) (15%) Find and sketch $P(j\omega)$, the continuous-time Fourier transform (FT) of $p(t)$.

(b) (10%) Consider the following two special cases: (i) $\tau \to 0$ (ii) $\tau = T$. For each case, write $p(t)$ and $P(j\omega)$ in a simplified form and provide a sketch for each.

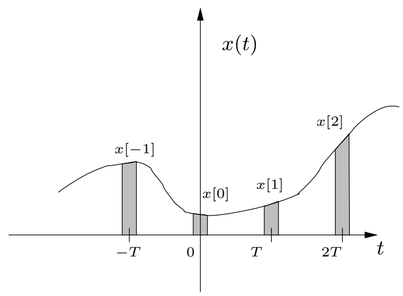
(c) (25%) The signal $p(t)$ goes through an LTI system with an impulse response $h(t)$ whose FT $H(j\omega)$ is given by



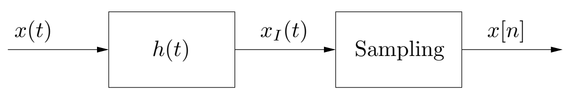
The resulting signal is given by $y(t) = p(t) * h(t)$. Find $y(t)$ in closed-form. (Hint: your answer should not include complex-valued numbers).

2. ($50\% + bonus$) The following question deals with non ideal sampling performed by short-time integration:

$$x[n] = \frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x(nT_s - t')dt'. \qquad (1)$$



This can be implemented in two steps:



First a short-time integration is applied $x_I(t) = x(t) * h(t)$, where $h(t)$ is given by

$$h(t) = \begin{cases} \frac{1}{\tau} & |t| < \frac{\tau}{2} \\ 0 & |t| \geq \frac{\tau}{2} \end{cases}$$

Then, $x_I(t)$ is sampled every T_s seconds via $x[n] = x_I(nT_s)$. For the rest of the question (unless otherwise stated) assume that $X(j\omega)$, the FT of $x(t)$, is band limited: $X(j\omega) = 0$ for $|\omega| \geq \omega_m$. Furthermore, assume $\tau < T$ and that sampling is performed above Nyquist rate.

(a) (5%) Verify that the block diagram above provides the same $x[n]$ as given by (1).

(b) (10%) Express $X_I(j\omega)$, the FT of $x_I(t)$, in terms of $X(j\omega)$.

(c) (10%) Express $X(e^{j\Omega})$, the DTFT of $x[n]$, in terms $X(j\omega)$.

(d) (15%) Design the perfect reconstruction of $x(t)$ from its samples $x[n]$ obtained by (1).

(e) (10%) What happens when $\tau \to 0$?

(f) (Bonus 10%) What happens when $\tau > T$? (Hint: express your answer in terms of perfect reconstruction).