Parallel connection:

\[ x(t) \rightarrow h_1(t) \rightarrow y_1(t) \]

\[ h(t) = h_1(t) + h_2(t) \]

When

\[ y(t) = h(t) \ast x(t) \]

Distribution property of convolution:

CT:

\[ x(t) \ast h_1(t) + x(t) \ast h_2(t) = x(t) \ast (h_1(t) + h_2(t)) \]

RT:

\[ x[\text{En}] \ast h[\text{En}] + x[\text{En}] \ast h_2[\text{En}] = x[\text{En}] \ast (h_1[\text{En}] + h_2[\text{En}]) \]

Cascade connection:

\[ x(t) \rightarrow h_1(t) \rightarrow Z(t) \rightarrow h_2(t) \rightarrow y(t) \]

\[ x(t) \rightarrow h_1(t) \ast h_2(t) \rightarrow y(t) \]

\[ h(t) \]
\[ z(t) = x(t) * h_1(t) = \int_{-\infty}^{\infty} x(u) h_1(t-u) \, du \]

\[ y(t) = z(t) * h_2(t) = \int_{-\infty}^{\infty} z(\tau) h_2(t-\tau) \, d\tau \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(u) h_1(t-u) h_2(t-\tau) \, du \, d\tau \]

let \( u = \tau - u \), \( du = d\tau \) for fixed \( u \), then

\[ y(t) = \int_{-\infty}^{\infty} x(u) \left( \int_{-\infty}^{\infty} h_1(u) h_2(t-u-v) \, dv \right) \, du \]

\[ = \int_{-\infty}^{\infty} x(u) \left( h_1(u) h_2(t-u) \right) \, du \]

\[ = \int_{-\infty}^{\infty} x(u) \left[ h(t-u) \right] \, du \]

\[ = x(t) * h(t) \]

where \( h(t) = h_1(t) * h_2(t) \)
**Associative property:**

\[ x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t) = \]

Same for DT signals.

**Commutative property:**

\[ h_1(t) * h_2(t) = h_2(t) * h_1(t) \]

**Ex:**

\[ h_1[n] = u[n] \]
\[ h_2[n] = u[n+2] - u[n] \]
\[ h_3[n] = s[n-2] \]
\[ h_4[n] = a^n u[n] \]

What is the overall \( h[n] \)?
\[ h[n] = (h_1[n] + h_2[n]) * h_3[n] + h_4[n] \]
\[ = u[n+2] * s[n-2] + \alpha u[n] \]
\[ = u[n] + \alpha u[n] \]
\[ = (1 + \alpha^n) u[n] \]

Ex: problem 2.03. An interconnection of LTI systems is depicted in the Figure below: \( h_1[n] = \left( \frac{1}{2} \right)^n u[n+2] \), \( h_2[n] = s[n] \), \( h_3[n] = u[n-1] \).

Find the impulse response of the overall system.

\[ h[n] = h_1[n] * (h_2[n] + h_3[n]) \]
\[ = \left( \frac{1}{2} \right)^n u[n+2] * (s[n] + u[n-1]) \]
\[ = \left( \frac{1}{2} \right)^n u[n+2] + \left( \frac{1}{2} \right)^n u[n+2] * u[n-1] \]
Now,
\[
\left( \frac{1}{2} \right)^n u[n+2] \ast u[n-1] = \sum_{k=-\infty}^{\infty} \left( \frac{1}{2} \right)^k u[k+2] u[n-1-k]
\]

Remember:
\[
\sum_{k=0}^{n} a^k = \frac{1 - a^{n+1}}{1 - a}
\]

\[
= \sum_{k=-\infty}^{n-1} \left( \frac{1}{2} \right)^k = \sum_{k=0}^{n-1} \left( \frac{1}{2} \right)^k + \left( \frac{1}{2} \right)^{-2} + \left( \frac{1}{2} \right)^{-1}
\]

\[
\Rightarrow n \geq -1
\]

\[
h[n] = \left( \frac{1}{2} \right)^n u[n+2] + \left[ 8 - \frac{1}{2} \right] u[n+1]
\]

Here is the graphical method:

\[
\begin{align*}
&\text{u}[n+2] \\
&\text{u}[n-1] \\
&\text{u}[-k-1] \\
&\text{u}[k-1]
\end{align*}
\]
\[ v \leq -1 + n \]

\[ -1 + n \leq -2 \]

\[ n \leq -1 \]
LT1 system properties and impulse response.

- Stability (BIBO)
- Memory
- Causality
- Linearity \( \Rightarrow \) LTI
- Time-invariance

For an LTI system:

\[ h(t) \quad \text{completely determine} \quad h[n] \xrightarrow{} \]

Thus, stability, memory, causality are related to \( h(t) \) and \( h[n] \).
a) For a given LTI system, if it is memoryless

Then memoryless \iff \quad h[k] = c \delta[k]

h[n] = h[n] \ast x[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]


(\Rightarrow) \quad h[0] = 0 \quad \text{for } \forall n \neq 0

\Rightarrow c \delta[n]