Chapter 2: Time-domain representation of LTI System

Focus:

H is a linear time-invariant (LTI) system. Analyzing the system given input find the system output.

Impulse response of an LTI system $H$: $h(t)$ completely characterize the system output can be obtained $y(t)$ for an arbitrary input signal $x(t)$ or $x[n]$.

* For $y(t)$ $|y[n]|$ related to $x(t)$ $|x[n]|$ and $h(t)$ $|h[n]|$.
Consider DT system.

Any signal \( x[n] \) can be expressed as a sum of time-shifted impulses:

\[
    x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k]
\]

Let's see: put in \( n=0 \) => \( x[0] = \sum_{k=-\infty}^{\infty} x[k] \delta[k] = x[0] \)
By employing LTI properties (superposition, scaling, shift-invariance), we will see that:

\[ s[n] \xrightarrow{H} h[n] \]

\[ x[k] s[n-k] \xrightarrow{H} x[k] h[n-k] \quad \text{(scaling and time-invariant)} \]

\[ x[n] = \sum_{k=-\infty}^{\infty} x[k] s[n-k] \xrightarrow{H} \sum_{k=-\infty}^{\infty} x[k] h[n-k] = y[n] \quad \text{(superposition)} \]

Now, we define:

**Convolution sum:**

\[ x[n] \ast h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \]

\[ y[n] = x[n] \ast h[n] \xrightarrow{H} y[n] \]

**Properties of convolution:**

- \( x[n] \ast h[n] = h[n] \ast x[n] \quad \text{(work it out yourself)} \)
- \( s[n] \ast h[n] = h[n] \)
- \( s[n-m] \ast h[n] = h[n-m] \)
- \( s[n-m] \ast h[n] = \sum_{k=-\infty}^{\infty} s[k-m] h[n-k] \)

\[ = h[n-m] \]
E: A system with input-output relationship as

\[ y[n] = x[n] + \frac{1}{2} x[n-1] \]  
(This is LTI system, convince yourselves)

1) What is the impulse response \( h[n] \)?

\[ h[n] = s[n] + \frac{1}{2} s[n-1] \]

(Replacing \( x[n] = s[n] \))

2) Find \( y[n] \) given

\[ x[n] = \begin{cases} 
4 & n = 1 \\
-2 & n = 2 \\
0 & \text{o.w.} 
\end{cases} \]

\[ x[n] = 2s[n] + 4s[n-1] - 2s[n-2] \]

\[ y[n] = x[n] * h[n] = (s[n] + \frac{1}{2} s[n-1]) * (2s[n] + 4s[n-1] - 2s[n-2]) \]


\[ = 2s[n] + 5s[n-1] - s[n-3] \]
\[
S_{[n-m]} \ast h[n] = h_{[n-m]}
\]

\[
S_{[n-1]} \ast S_{[n]} = S_{[n-1]}
\]

\[
S_{[n-1]} \ast S_{[n-1]} = S_{[n-2]}
\]