Outline

• Exam 1 Review

• Cameras

• Pinhole camera
Problem 2: Harris Corner Detector

\[ E(x, y) = [u \ v] M(x, y) \begin{bmatrix} u \\ v \end{bmatrix} \]

\[ M(x, y; \sigma) = w(x, y; \sigma) * \begin{bmatrix} I_x^2(x, y) & I_x(x, y)I_y(x, y) \\ I_x(x, y)I_y(x, y) & I_y^2(x, y) \end{bmatrix} \]

\[ M(x, y; \sigma) = \begin{bmatrix} w(x, y; \sigma) * I_x^2(x, y) & w(x, y; \sigma) * I_x(x, y)I_y(x, y) \\ w(x, y; \sigma) * I_x(x, y)I_y(x, y) & w(x, y; \sigma) * I_y^2(x, y) \end{bmatrix} \]
Problem 2: Hessian Detector

\[ E(x, y) = [u \ v] M(x, y) \begin{bmatrix} u \\ v \end{bmatrix} \]

\[ M(x, y; \sigma) = w(x, y; \sigma) * \begin{bmatrix} I_{xx}(x, y) & I_{xy}(x, y) \\ I_{xy}(x, y) & I_{yy}(x, y) \end{bmatrix} \]

\[ M(x, y; \sigma) = \begin{bmatrix} w_{xx}(x, y; \sigma) * I(x, y) & w_{xy}(x, y; \sigma) * I(x, y) \\ w_{xy}(x, y; \sigma) * I(x, y) & w_{yy}(x, y; \sigma) * I(x, y) \end{bmatrix} \]
Problem 2: Harris/Hessian Detector

Harris/Hessian Detector is NOT Scale Invariant

Source: L. Fei-Fei
Problem 3: Eigenvalues and Eigenvectors of $M$

- Eigenvectors of $M$ -- directions of the largest change of $E(x,y)$
- Eigenvalues of $M$ -- the amount of change along eigenvectors

$$E(x, y) = [u \ v] M(x, y) \begin{bmatrix} u \\ v \end{bmatrix}$$
Problem 3: Eigenvalues of M

- **“Corner”**
  \( \lambda_1 \) and \( \lambda_2 \) are large,
  \( \lambda_1 \sim \lambda_2 \);
  \( E \) increases in all directions

- **“Flat” region**

- **“Edge”**
  \( \lambda_1 \gg \lambda_2 \)
Problem 3: Affine Normalization

Ellipse in image 1 \( \implies \) Unit circle in image 2

\[ p^T M^{-1} p = 1 \]

\[ q^T I^{-1} q = 1 \]

where

\[ q = T p \]

\[ T = ? \]
Problem 3: Affine Normalization

Ellipse in image 1 \[ \iff \] Unit circle in image 2

\[ p^T M^{-1} p = 1 \]

\[ q^T I^{-1} q = 1 \]

where

\[ q = Tp \]

\[ T = ? \]
Problem 3: Affine Normalization

Ellipse in image 1 $\Rightarrow$ Unit circle in image 2

$p^T M^{-1} p = 1$

$q^T I^{-1} q = 1$

where

$q = Tp$

$I = TMT^T$

$I = T\Phi\Lambda\Phi^T T^T$

$I = T\Phi\Lambda^{\frac{1}{2}}(T\Phi\Lambda^{-\frac{1}{2}})^T$
Problem 3: Affine Normalization

Ellipse in image 1 $\equiv$ Unit circle in image 2

$$p^T M^{-1} p = 1$$

$$q^T I^{-1} q = 1$$

where

$$q = Tp$$

$$T = \Phi^T \Lambda^{-\frac{1}{2}}$$
Problem 4: SIFT Descriptor

128-D vector = (4x4 blocks) x (8 bins of histogram)

---

gradient angles of 16x16 patch centered at the point

accumulated histogram of gradient angles of 4x4 subpatch

The figure illustrates only 8x8 pixel neighborhood that is transformed into 2x2 blocks, for visibility
Problem 4: DAISY

3. Compute the DAISY descriptor of a keypoint at a number of pixel locations \((x_i, y_i)\) in the vicinity of that keypoint, where each location has its corresponding \(\Sigma_i\) as shown in the figure.

\[
D = \left[ 
M_{\Sigma_0}^{o1}(x_0, y_0), M_{\Sigma_0}^{o2}(x_0, y_0), \ldots, M_{\Sigma_0}^{o8}(x_0, y_0), \\
M_{\Sigma_1}^{o1}(x_1, y_1), M_{\Sigma_1}^{o2}(x_1, y_1), \ldots, M_{\Sigma_1}^{o8}(x_1, y_1), \\
\ldots \\
M_{\Sigma_i}^{o1}(x_i, y_i), M_{\Sigma_i}^{o2}(x_i, y_i), \ldots, M_{\Sigma_i}^{o8}(x_i, y_i), \ldots \right]^T
\]
Problem 6: Beam-Angle Descriptor

- Shape = Sequence of points along the shape
- Each point is characterized by a descriptor:
  - Beam-angle
Problem 7

\[
\max_X \text{tr}(\Psi^T X)
\]

subject to:

\[\forall u \in V, \forall u' \in V', \ x_{uu'} \in [0, 1]\]

\[\forall u, \ \sum_{u'} x_{uu'} = 1\]

\[\forall u', \ \sum_u x_{uu'} = 1\]

one-to-one matching
Rayleigh-Ritz Theorem

\[ \max_X \frac{X^T \Psi X}{X^T X} = \lambda_{\text{max}} \]

subject to:

\[ \forall v \in V, \forall v' \in V', x(v, v') \in [0, 1] \]
number of students

points
Imaging Process
How to Design a Camera?

Do we get a reasonable image if we put a film in front of an object?

source: S. Savarese
Pinhole Camera

- The barrier blocks off most of the rays
- This reduces blurring
- Aperture = Opening of the pinhole

source: S. Savarese
Shrinking the Aperture...

pinhole too big:  
  bright  
  blurred

pinhole right size:  
  dark  
  crisp

pinhole too small:  
  dark  
  diffraction blur
The lens focuses light onto the film

source: S. Savarese
Combining Lenses...

source: S. Savarese
Issues with Lenses: Chromatic Aberration

different refractive indices for different light wavelengths

source: S. Savarese
Issues with Lenses: Radial Distortion

source: S. Savarese
Digital Cameras

- 2D array of light-sensitive diodes
- The diodes convert photons to electrons
- Two common types:
  - Charged coupled device (CCD)
  - CMOS
Issues with Digital Cameras

interlaced scanning

1st field: Odd field
2nd field: Even field
One complete frame using interlaced scanning

progressive scanning

One complete frame using progressive scanning

source: S. Seitz
Issues with Digital Cameras

interlaced scanning produces blur

source: S. Seitz
Issues with Digital Cameras

progressive scanning produces blur
Pinhole Camera -- Perspective Projection

we will often use the virtual image
Properties of Images Acquired by Pinhole Cameras

distant objects are smaller

source: S. Seitz
parallel lines in 3D world meet at a point in the image
Vanishing Points

• Vanishing point = Intersection of parallel lines

• Distinct sets of parallel lines have distinct vanishing points
Identifying the Horizon in Images
Identifying the Horizon in Images
Identifying the Horizon in Images
Identifying the Horizon in Images
Properties of Perspective Projection

• Points project to points
• Lines project to lines
• Planes project to regions
• Angles are not preserved