Chebyshev's Inequality:

\[
P( |X - m_x| > k \sigma_x ) \leq \frac{1}{k^2}
\]

\[
P( |X - m_x| > b ) \leq \frac{\sigma_x^2}{b^2}
\]

\[
E[X] = \int_{-\infty}^{\infty} x f(x) \, dx = \int_{-\infty}^{0} x f(x) \, dx + \int_{0}^{\infty} x f(x) \, dx
\]

\[
= \int_{0}^{a} x f(x) \, dx + \int_{a}^{\infty} x f(x) \, dx
\]

\[
\geq a \int_{a}^{\infty} f(x) \, dx = a P(X \geq a)
\]

\[
\Rightarrow P(X \geq a) \leq \frac{E(X)}{a}
\]
Note that we assume $x > 0$,

$$Y = (X - m_x)^2$$

using Markov’s inequality,

$$P(Y > b^2) \leq \frac{E(Y)}{b^2}$$

$$P((X - m_y) > b) \leq E\left(\frac{(X - m_x)^2}{b^2}\right) = \frac{s_x^2}{b^2}$$
Ex:  The mean resistance of a shipment of resistors is 100 Ω.
It is also known that variance $\sigma^2_x$ is 16 (Ω)$^2$.

Find a lower bound for $P(88 \leq x \leq 112)$.

\[
P(88 \leq x \leq 112) \geq \frac{1}{\sqrt{12}}
\]

\[
P(88 \leq x \leq 112) = P(88-100 \leq x-100 \leq 12)
\]

\[
= P\left( |x-100| \leq 12 \right) = 1 - P(|x-100| > 12)
\]

\[
\geq 1 - \frac{8^2}{(12)^2} = 1 - \frac{16}{144} = \frac{8}{9}
\]
TWO RANDOM VARIABLES:

Remember: \[ X \leftrightarrow \xi_X(x) \leftarrow \xi_X(x) \]

\[ (X, Y) \leftrightarrow \xi_{XY}(x, y) \leftarrow \xi_{XY}(x, y) \]
\[ P(X \leq x^*, Y \leq y^*) = F_{XY}(x^*, y^*) \]
Properties of joint CDF \( F_{xy}(x,y) \):

1. \( F_{xy}(-\infty, y) = F_{xy}(x, -\infty) = 0 \)
   \( P(x \leq -\infty, y) = P(x, y \leq -\infty) = 0 \)

2. \( F_{xy}(\infty, \infty) = 1 \)
   \( P(x \leq \infty, y \leq \infty) = 1 \)

3. \( F_{xy}(x, \infty) = F_x(x) \) \( \leq \) marginal CDF
   \( F_{xy}(\infty, y) = F_y(y) \) \( \geq \)
   \( P(x \leq \infty, y) = P_y(y \leq y) = F_y(y) \)