Definition: Given a sample space $S$, a probability defined on each event of $S$, a random process $X(t, \xi)$ is a time function assigned to each outcome $\xi \in S$.

Ex: Let's flip a fair coin twice. Suppose that for each outcome, we define

- $X(t, HT) = \sin(t)$
- $X(t, TH) = \sin(2t)$
- $X(t, TT) = \sin(3t)$
- $X(t, HH) = \sin(4t)$
\[ P(X(t_0) \leq a) . \]
\[ P(X(0) \leq 0.5) = 1 \]
\[ P(X(\frac{\pi}{2}) \leq 0.5) = \frac{3}{4} \]

\[ F_X(x) = \begin{cases} 
 0 & \text{if } \sin(\frac{\pi}{2}) = 0 \\
 1 & \text{if } \sin(2\pi) = 1 \\
 \frac{3}{4} & \text{if } \sin(\frac{3\pi}{2}) = 0 \\
 0 & \text{if } \sin(4\pi) = 0 
\end{cases} \]
Distribution of Random Processes.

Definition: Let $X(t) = X(t, s)$. If we sample the random process at time $t_i$, $X(t_i)$ is a random variable which can be described in terms of its CDF $F_{X(t)}(x) = F_X(x; t_i) = P(X(t_i) \leq x)$.

If $F_X(x; t_i)$ is differentiable w.r.t. $x$, the pdf of $X(t_i)$ is described by

$$f_X(x; t_i) = \frac{dF_X(x; t_i)}{dx}$$

Example: Let's find the distribution of the random process of the previous example.
Classification of random processes.

1. Continuous time, continuous amplitude
2. Continuous time, discrete amplitude
3. Discrete time, continuous amplitude
4. Discrete time, discrete amplitude

Ex:

This is a continuous time, discrete amplitude random process where $T_b$ is the bit duration, $t_0$ is a random time delay which is uniformly distributed in $(0, T_b)$ at any given $t$, the amplitude is either $+A$ or $-A$ with probability $\frac{1}{2}$. 
\[ p(X(t_i) = \sin(t_i)) = \frac{1}{4} = p(X(t_i) = -\sin(t_i)) = p(X(t_i) = \sin(\pi t_i)) = p(X(t_i) = -\sin(\pi t_i)) \]

**Definition:** If we sample the random process at times \( t_1 \) and \( t_2 \), then we have two random variables \( X(t_1) \) and \( X(t_2) \)

\[ F_X(x_1, x_2; t_1, t_2) = p(X(t_1) \leq x_1; X(t_2) \leq x_2) \]

\[ f_X(x_1, x_2; t_1, t_2) = \frac{d^2}{dx_1 dx_2} F_X(x_1, x_2; t_1, t_2) \]

**Definition:** A random process \( X(t) \) is stationary if it has the same nth order distribution function as \( X(t+\tau) \), \( \forall \tau \), i.e.

\[ F_X(x_1, \ldots, x_n; t_1, \ldots, t_n) = F_X(x_1, \ldots, x_n; t_1 + \tau, \ldots, t_n + \tau) \]