\[ P(A \cup B) = P(A) + P(B) - P(\overline{A} \cap \overline{B}) \]

\[ P(A \cup B) = P(\overline{A} \cup \overline{B}) \]

\[ P(B) = P(\overline{A} \cup \overline{B}) + P(\overline{A} \cap \overline{B}) \]

\[ \Rightarrow P(A \cup B) = P(A) + P(B) - P(\overline{A} \cap \overline{B}) \]

\[ P(\emptyset) = 0 \]
Consider an experiment of flipping a coin twice.

Define the events as follows:

\[ A = \text{a head followed by a head} \]
\[ B = \text{a head followed by a tail} \]
\[ C = \text{a head occurs at the first toss} \]

\[ C = A \cup B \]

\[ P(C) = P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

\[ \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2} \]

\[ = \frac{1}{2} \]
\[ P(AnBnC) = P(AnB) + P(A\cup B) - P(A\cup B\cup C) \]

\[ P(A\cup B\cup C) = P(\overline{A}\cup \overline{B}\cup \overline{C}) \]

Now \[ \omega = A\cup B \]

\[ P(A\cup B\cup C) = P(\overline{D}\cup \overline{C}) = P(D) + P(C) - P(D\cap C) \]

\[ P(A\cup B) + P(C) = P(A\cup B\cup C) \]

\[ P(A) + P(B) - P(AnB) + P(C) = P(A\cup B\cup C) \]
\((A \cap B \cap C) = \triangle\)

\((A \cap C \cap \overline{B})\)

\((A \cap \overline{B}) \cup (B \cap \overline{A})\)

\(\overline{A \cap B}\)
Conditional Probability and Independence.

Conditional probability: Suppose we have prior information of the occurrence of an event B. Then, we would like to know what the probability of occurrence of another event A, given that event B has occurred.

If \( P(B) \neq 0 \)

then \( P(A|B) = \frac{P(A \cap B)}{P(B)} \)
1. \(\emptyset = (A \cap B)\), then \(P(A|B) = \frac{P(\emptyset)}{P(B)} = 0\) \(\frac{P(B)}{P(B)} = 0\)

2. \(B \subseteq A \Rightarrow A \cap B = B\) and

\[ P(A|B) = \frac{P(B \cap A)}{P(B)} = \frac{P(B)}{P(B)} = 1 \]
ex: A pair of fair dice is rolled once. Suppose that the two numbers that occur are different.
What is the probability that the sum is 7.

Solution: \( A \) = event that two numbers are different
\( B \) = event that the sum is 7.

\[
P(B | A) = \frac{P(A \cap B)}{P(A)}
\]

\[
P(A) = \frac{30}{36} = \frac{5}{6}
\]

There are 36 possible outcomes.
6 outcomes in which the numbers are different.

\[
P(B) = \frac{6}{36} = \frac{1}{6}
\]
\[ P(AnB) = P(B) \quad \text{because} \quad BCA \]

\[ = \frac{6}{36} \]

\[ \Rightarrow P(B|A) = \frac{P(AnB)}{P(A)} = \frac{P(B)}{P(A)} = \frac{6}{36} = \frac{36}{30} = \frac{4}{5} \]