1. (15pts) Let $A$, $B$, and $C$ be an events such that $P(A) + P(B) + P(C) = 1$. Are $A$, $B$, and $C$ mutually exclusive events? Provide a mathematical justification for either "Yes" or "No" answers.

2. (15pts) A deck of cards is shuffled thoroughly. You randomly pick a card, observe whether the card is a heart, a diamond, a spade, or a club, then put it back, and shuffle again. Let $N$ be a random variable denoting the number of times that you pick the cards until you observe 5 hearts. Find the probability mass function $P_N(n)$.

3. (30pts) To communicate one bit of information reliably, cellular phones transmit the same binary symbol five times. Thus the information "zero" is transmitted as 00000 and "one" is 11111. The receiver guesses the information "zero" was sent if it receives more "0"s than "1"s. Otherwise, it guesses the information "one" was sent. Assuming binary symbol errors (bit flip) during the transmission occur independently with probability $q$,

(a) Find the probability that the receiver makes a wrong guess.
(b) Let $X$ be a random variable denoting the number of consecutive wrong guesses that the receiver makes in a row before making a right guess, find $F_X(x)$.

4. (40pts)
Suppose traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed in such a way that given a driver encounters a red or green at the first traffic light, with probability 0.8 a driver will find the second traffic light to have the same color, i.e., $P(G_2|G_1) = P(R_2|R_1) = 0.8$, where $G_i$ and $R_i$ denote the events that the $i$th traffic light is green and red, respectively. Assuming the first light is equally likely to be red or green,

(a) What is the probability $P(G_2)$, the probability that the second light is green?

(b) What is $P(W)$?, the probability that you wait for at least one light?
EXTRA SPACE: