CS 440: Database Management Systems
Inference Rules

- IR1 (reflexive rule): If $X \supseteq Y$, then $X \rightarrow Y$.
- IR2 (augmentation rule): $\{X \rightarrow Y\} \models XZ \rightarrow YZ$
- IR3 (transitive rule): $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$
- IR4 (projective rule): $\{X \rightarrow YZ\} \models X \rightarrow Y$
- IR5 (additive rule): $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$
- IR6 (pseudotransitive rule): $\{X \rightarrow Y, WY \rightarrow Z\} \models WX \rightarrow Z$
Closure

• Determining $X^+$, the Closure of $X$ under $F$
  - Algorithm 16.1
    
    $X^+ := X$;
    
    Repeat
    
    Old$X^+$ := $X^+$;
    
    For each FD $Y \rightarrow Z$ in $F$ do
    
    If $X^+ \supseteq Y$ then $X^+ := X^+ \cup Z$;
    
    Until ($X^+ = \text{Old}X^+$);
  
  - Example

$R = \{\text{SSN, EName, PNumber, Pname, Plocation, Hours}\}$

$F = \{\text{SSN} \rightarrow \text{EName }, \text{PNumber} \rightarrow \{\text{PName, Plocation}\}, \{\text{SSN, PNumber}\} \rightarrow \text{Hours}\}$
Equivalence

- Definition – A set of functional dependencies $F$ is said to cover another set of functional dependencies $E$ if every FD in $E$ is also in $F^+$; that is, if every dependency in $E$ can be inferred from $F$; alternatively, we can say that $E$ is covered by $F$.

- Definition – Two sets of functional dependencies $E$ and $F$ are equivalent if $E^+ = F^+$. Therefore, equivalence means that every FD in $E$ can be inferred from $F$, and every FD in $F$ can be inferred from $E$; that is, $E$ is equivalent to $F$ if both the conditions – $E$ covers $F$ and $F$ covers $E$ – hold.
Example

- F={A→C, AC→D, E→AD, E→H}
- G={A→CD, E→AH}
- Show these two sets are equivalent.
Minimal Cover

- It is always possible to find a dependency-preserving decomposition \( D \) with respect to \( F \) such that each \( R_i \) in \( D \) is in 3NF.

- Algorithm:
  - Need to be able to find minimal cover of a set of functional dependencies
Minimal Cover

- Finding a minimal cover $F$ for a set of functional dependencies $E$
  - Step 1: $F := E$
  - Step 2: replace each $X \rightarrow \{A_1, A_2, \ldots, A_n\}$ by $\{X \rightarrow A_1, X \rightarrow A_2, \ldots, X \rightarrow A_n\}$
  - Step 3: For each function dependency $X \rightarrow A$ in $F$
    - For each attribute $B$ that is an element of $X$
      - If $\{\{F - \{X \rightarrow A\}\} \cup \{(X - \{B\}) \rightarrow A\}\}^+ = F^+$
        replace $X \rightarrow A$ with $(X - \{B\}) \rightarrow A$
    - Step 4: For each remaining FD in $F$
      - If $\{F - \{X \rightarrow A\}\}^+ = F^+$ then remove $X \rightarrow A$
Minimal Cover

- Example:
  \[ E = \{ B \rightarrow A, D \rightarrow A, AB \rightarrow D \} \]

- Step 2: no action

- Step 3:
  \[ \{ B \rightarrow A, D \rightarrow A, AB \rightarrow D \}^+ \]
  \[ = \{ B \rightarrow A, D \rightarrow A, B \rightarrow D \}^+ \]
  \[ = \{ D \rightarrow A, B \rightarrow D \}^+ \]

- The minimal cover of \( E \) is \{D->A, B->D\}.
Find a Key $K$ for $R$, given a set $F$ of FDs.

- Set $K := R$.
- For each attribute $A$ in $K$
  - Compute $(K-A)^+$ with respect to $F$;
  - If $(K-A)^+$ contains all the attributes in $R$, then set $K := K \setminus \{A\}$
Key/Minimal Set Cover...

- \( R = \{A, B, C, D, E, F, G, H, I, J\} \)
- \( F = \{ \{A, B\} \rightarrow \{C\}, \{A\} \rightarrow \{D, E\}, \{B\} \rightarrow \{F\}, \{F\} \rightarrow \{G, H\}, \{D\} \rightarrow \{I, J\} \} \)
- Determine a key for \( R \).
- Find the minimum set of dependencies \( G \) that is equivalent to \( F \).
Another example…

- $R = \{A, B, C, D, E, F, G, H, I, J\}$
- $G = \{\{A, B\} \rightarrow \{C\}, \{B, D\} \rightarrow \{E, F\}, \{A, D\} \rightarrow \{G, H\}, \{A\} \rightarrow \{I\}, \{H\} \rightarrow \{J\}\}$.

- Determine a key for $R$.
- Find the minimum set of dependencies $G$ that is equivalent to $F$. 