1. Prove the following statement by induction.

\[ \sum_{i=0}^{k} 2^i = 2^{k+1} - 1 \]

Please clearly state the following for your proof:

**Base case (2pts):**

\( k = 0: \)

left side = \( 2^0 = 1 \)

right side = \( 2^1 - 1 = 1 \)

**Inductive assumption (2pts):**

Assume that

\[ \sum_{i=0}^{t} 2^i = 2^{t+1} - 1 \text{ for } t = 0, 1, ..., k - 1 \]

Or alternatively Assume that

\[ \sum_{i=0}^{k-1} 2^i = 2^k - 1 \]

**Inductive step (4pts):**

\[ \sum_{i=0}^{k} 2^i = \sum_{i=0}^{k-1} 2^i + 2^k \]

Using the inductive hypothesis:

\[ \sum_{i=0}^{k} 2^i = (2^k - 1) + 2^k = 2^k + 2^k - 1 = 2^{k+1} - 1 \]
This completes the proof.

2. Below is an incomplete pseudo-code for an algorithm that finds the median of two sorted arrays \(a[1, ..., n]\) and \(b[1, ..., n]\). It compares the medians of the two arrays, and based on the comparison shrinks the arrays by half.

```plaintext
function median2(a[1, ..., n], b[1, ..., n])
1. if \(n \leq 2\): Explicitly find the median and return it;
2. \(a_m \leftarrow \) Median of \(a\)
3. \(b_m \leftarrow \) Median of \(b\)
4. if \(a_m == b_m\): return \(a_m\)
5. if \(a_m > b_m\):
6. return \(\text{A}\)
7. if \(a_m < b_m\):
8. return \(\text{B}\)
```

(3 pts each) Complete the pseudo-code by filling in lines 6 and 8, each with one of the four choices provided below:

A. median2(a[1, ..., \([n/2]\)], b[1, ..., \([n/2]\)])
B. median2(a[1, ..., \([n/2]\)], b[\([n/2] + 1, \ldots, n\)])
C. median2(a[\([n/2] + 1, \ldots, n\)], b[\([n/2] + 1, \ldots, n\)])
D. median2(a[\([n/2] + 1, \ldots, n\)], b[1, \ldots, \([n/2]\)])