As we saw in class, the following problem is NP-complete.

**Hamiltonian Cycle**

**Input:** Undirected graph \( G = (V, E) \)

**Question:** Does \( G \) has a cycle that visits each vertex in \( V \) exactly once?

Consider Hamiltonian Cycle restricted to graphs in which every vertex has at most 2 edges. Call this problem Hamiltonian Cycle-2.

a (4 pts) Prove that Hamiltonian Cycle-2 is in NP.

Given a proposed solution, which is a permutation of the nodes of the graph \( v_1, v_2, ..., v_n \), we simply need to check if there exists an edge between \( v_i \) and \( v_{i+1} \) for \( i = 1, ..., n - 1 \) and between \( v_n \) and \( v_1 \). If any edge is missing, it is an invalid solution, otherwise it is valid. This can clearly be done in linear time, showing that the problem is in NP.

b (6 pts) What is wrong with the following proof of NP-completeness for Hamiltonian Cycle-2?

We know that the Hamiltonian Cycle problem is NP-complete, so it is enough to present a reduction from Hamiltonian Cycle-2 to Hamiltonian Cycle. Given a graph \( G \) with vertices of degree at most 2, the reduction simply leaves the graph unchanged to be the input for Hamiltonian Cycle. The answer to both problems would clearly be identical. This proves the correctness of the reduction, thus Hamiltonian Cycle-2 is NP-complete.

The direction of the reduction is wrong. We need to reduce from HC to HC-2. To be complete, you should also show that HC-2 is in NP.