1. Prove the following statement.
   When using the union-by-rank data structure, any root node of rank $k$ has at least $2^k$ nodes in the tree.

2. Consider the Change Problem in Austria. The input to this problem is an integer $L$. The output should be the minimum cardinality collection of coins required to make $L$ shillings of change (that is, you want to use as few coins as possible). In Austria the coins are worth 1, 5, 10, 20, 25, 50 Shillings. Assume that you have an unlimited number of coins of each type. Formally prove or disprove that the greedy algorithm (that takes as many coins as possible from the highest denominations) correctly solves the Change Problem. So for example, to make change for 234 Shillings the greedy algorithms would take four 50 shilling coins, one 25 shilling coin, one 5 shilling coin, and four 1 shilling coins.

3. Textbook 5.8

4. Textbook 5.14

5. Textbook 5.15

6. Textbook 5.16