CS 536: Introduction to Graphical Models  
Winter 2015  
Assignment #3

Out: Friday, Jan 23, 2015  
Due: In class, Monday, Feb 2, 2015  
Total marks: 45

1. (Exercise 4.1 in the book) Complete the analysis of example 4.4, showing that the distribution $P$ defined in the example does not factorize over $\mathcal{H}$. (Hint: Use a proof by contradiction). If you read Theorem 4.1 to Example 4.4 on pages 115-116, it will really help with this question. [10 points]

2. (Slightly modified version of Exercise 4.13 in the book) Show that we can convert any Gibbs distribution with factors represented as tables into a log-linear model as defined in definition 4.15. [5 points]

3. (Exercises 4.14 in the book). The Markov blanket of a node $X$ in a Bayesian network $G$, denoted $\text{MB}_G(X)$, is defined to be the nodes consisting of $X$’s parents, $X$’s children, and other parents of $X$’s children. Show the following:

   a) For any variable $X$, let $W = X - \{X\} - \text{MB}_G(X)$. Then $d$-$\text{sep}_G(X; W | \text{MB}_G(X))$ [10 points]

   b) The set $\text{MB}_G(X)$ is the minimal set for which this property holds. [10 points]

4. (Exercise 4.18 in the book) Let $G$ be a Bayesian network structure and $\mathcal{H}$ a Markov network structure over $X$ such that the skeleton of $G$ is precisely $\mathcal{H}$. Prove that if $G$ has no immoralities, then $\mathcal{I}(G) = \mathcal{I}(\mathcal{H})$. [10 points]