CS 536: Introduction to Graphical Models Practice Midterm

Note: Your actual midterm is 1 hr long. This practice midterm was from a class in which the midterm was 90 mins long.

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<th>Section</th>
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<tr>
<td>Inference</td>
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<td>Independence/Graph relationships</td>
<td>/ 35</td>
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<td>Total</td>
<td>/ 45</td>
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</table>
Section I: Inference [10 points]
1. You will compute probabilities using the Bayesian Network below. The conditional probability tables for \( X_1 \) and \( X_2 \) are shown below. The tables for \( Y_1 \) and \( Y_2 \) are identical and are shown in the \( P(Y_i|X_i) \) table. You may assume all variables are Boolean.

\[
\begin{array}{c|c}
X_1 & P(X_1) \\
\hline
\text{False} & 0.5 \\
\text{True} & 0.5 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
X_1 & X_2 & P(X_2|X_1) \\
\hline
\text{False} & \text{False} & 0.6 \\
\text{False} & \text{True} & 0.4 \\
\text{True} & \text{False} & 0.2 \\
\text{True} & \text{True} & 0.8 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
X_i & Y_i & P(Y_i|X_i) \\
\hline
\text{False} & \text{False} & 0.9 \\
\text{False} & \text{True} & 0.1 \\
\text{True} & \text{False} & 0.1 \\
\text{True} & \text{True} & 0.9 \\
\end{array}
\]

a) \( P(X_1 = \text{False}|Y_1 = \text{False}) \) [4 points]
b) \( P(Y_2=\text{False}|X_1=\text{False}, Y_1=\text{False}) \) [6 points]
Section II: Independence/Graph relationships [35 points]

1. Suppose there are four variables X, Y, Z, W. The possible values that each can take are:
   \[X = \{x_1, x_2\}\quad Y = \{y_1, y_2\}\]
   \[Z = \{z_1, z_2\}\quad W = \{w_1, w_2, w_3\}\]

Let the joint distribution \(P(X,Y,Z,W)\) [Note: this is a joint, not a conditional] be given by the following table:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>W</th>
<th>(P(X,Y,Z,W))</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>y1</td>
<td>z1</td>
<td>w1</td>
<td>1/3</td>
</tr>
<tr>
<td>x1</td>
<td>y2</td>
<td>z2</td>
<td>w2</td>
<td>1/3</td>
</tr>
<tr>
<td>x2</td>
<td>y2</td>
<td>z1</td>
<td>w3</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>All other configurations</td>
<td>0</td>
</tr>
</tbody>
</table>

a) Show that the graph \(G\) given below is a minimal I-map of \(P\). [5 points]
b) Show that the joint probability distribution $P$ cannot be expressed as a product of functions on the cliques of $G$ (Hint: Finding a single counterexample will do). You may assume that the clique potentials can be obtained by marginalizing over the joint probability distribution. [5 points]

2. Suppose $P$ is a probabilistic model that can be represented by both a Bayesian network and a Markov network.
   a) What specific characteristic of a probabilistic model allows it to be represented by both a Bayesian network and a Markov network? [2 points]

   b) Why would you prefer to model $P$ using a Bayesian network rather than a Markov network? Give 2 reasons. [4 points]

   c) Why would you prefer to model $P$ using a Markov network rather than a Bayesian network? Give 1 reason. [2 points]
3. Suppose we try to represent the Markov network in (a) using the Bayesian networks in (b) and in (c). We already know that this doesn’t work. Explain why the two Bayesian networks do not represent the same conditional independencies as the Markov network in (a). [5 points]