I. HOMEWORK 4

A. Problem 1

Let r.v. \( X \) be defined by its probability density function as follows
\[
f(x) = \begin{cases} 
\lambda xe^{-x}, & x \geq 0; \\
0, & \text{otherwise}.
\end{cases}
\]

Please, 1) determine parameter \( \lambda \); 2) find c.d.f. of \( X \); 3) find \( P(X > 9) \); 4) find \( P(0 \leq X < 3) \).

B. Problem 2

Consider an interval \([a, b]\), where \( a < b \). Let \( X \) be a continuous r.v. with density function
\[
f(x) = \begin{cases} 
\frac{1}{b-a}, & x \in [a, b]; \\
0, & \text{otherwise}.
\end{cases}
\]

Random variable \( X \) is said to be an uniform random variable over \([a, b]\). Please, 1) check that \( f(x) \) is indeed a probability density function, sketch \( f(x) \); 2) find c.d.f of \( X \) and sketch it.

C. Problem 3

Suppose \( Z \) is a random number chosen uniformly from the interval \([-3, 6]\). What is the probability that the quadratic equation
\[x^2 + Zx + 4 = 0\]
has no real root?

D. Problem 4

Based on the last ten year statistics, an approximate estimate of the daily average temperature (in degrees in Fahrenheit) in Corvallis on April 26 is \( N(54, 6^2) \). Assume that the temperatures in Corvallis this year (April 26, 2015) will follow the same distribution. 1) Calculate the probability that the temperature will exceed 66 degrees. 2) Define an extreme temperature as either less than 32 or greater than 80. Calculate the probability of extreme temperature.

E. Problem 5

Assume that male height in inches is distributed as \( N(70, 32) \). 1) Find the probability that a male chosen at random is taller than 60 inches. 2) Find the probability that a male chosen at random is taller than LeBron James (79.92 inches).