I. Homework 5

A. Problem 1

Roll one fair die. Let r.v. X represent the outcome of this experiment. Find $E[X]$ and $Var[X]$. 

B. Problem 2

Let r.v. X be defined by its probability density function as follows

$$f(x) = \begin{cases} 
2x, & 0 \leq x \leq 1; \\
0, & \text{otherwise.}
\end{cases}$$

Please find $E[X]$, second moment $m_2$, and second central moment $\mu_2 = Var[X]$. 

C. Problem 3

Let $X$ be continuous r.v. with density function $f(x)$. Prove 1) $E[aX + b] = aE[X] + b$ 

2) $Var[aX + b] = a^2Var[X]$. 

D. Problem 4

Let $X$ be nonnegative r.v. with $E[X] = 3$ and $SD[X] = 1$. Find the upper bounds on the following probabilities: 1) $P(X \geq 100)$ and 2) $P(|X - 3| \geq 10)$. (You may want to use Markov’s and Chebyshev’s inequalities.)

E. Problem 5

Let $X$ be a uniform random variable on interval $[a, b]$. Please find $E[X]$ and $Var[X]$. 