1. Consider a Wiener filtering problem characterized as follows: The correlation matrix \( R \) of the input vector \( X[n] \) is:

\[
R = \begin{pmatrix}
1 & 0.5 \\
0.5 & 1
\end{pmatrix}
\]

The cross-correlation vector \( p \) between the \( X[n] \) and the desired response (output) \( D[n] \) is:

\[
p = \begin{pmatrix}
0.5 \\
0.25
\end{pmatrix},
\]

and \( \sigma_d^2 = 1 \).

(a) Find the optimal tap weights \( (w_i) \) of the Wiener filter.

(b) What is the minimum mean-squared error produced by this Wiener filter?

2. Let \( X[n] = 0.9X[n-1] + V[n] \) be a random sequence. \( V[n] \) is an i.i.d random sequence with mean \( \mu_V = 0 \) and variance \( \sigma_V^2 = 0.1 \), and is independent with \( X[n] \). Assume \( X[n] \) is wide-sense stationary.

(a) Find \( R_{XX}[m] \).

(b) Find the one-tap optimal Wiener filter to predict \( X[n] \) from \( X[n-1] \). Does your answer make sense?

(c) Find the two-tap optimal Wiener filter to predict \( X[n] \) from \( X[n-1] \) and \( X[n-2] \). Compare with answer from (b), does your answer make sense?

3. Problem 8.26

4. Problem 8.28

5. Problem 8.31

6. Problem 8.32

7. Problem 8.35

8. Problem 8.37