1. The continuous-time signal 
\[ x_c(t) = \cos(4000\pi t) \]

is sampled with a sampling period \( T \) to obtain the discrete-time signal 
\[ x[n] = \cos\left(\frac{\pi n}{3}\right) \]

(a) Determine a choice for \( T \) consistent with this information.
(b) Is your choice for \( T \) in part (a) is unique? If so, explain why. If not, specify another choice of \( T \) consistent with the information given.

2. (a) Continuous signal 
\[ x_c(t) = \sin(20\pi t) + \cos(40\pi t) + \sin(60\pi t) \]

is used as the input for an ideal C/D converter as shown in Fig. 1 with the sampling period \( T = \frac{1}{100} \text{sec.} \)

Find the resulting discrete-time signal \( x[n] \).

(b) The continuous-time signal 
\[ x_c(t) = \frac{\sin(10\pi t)}{10\pi t} \]

is sampled with a sampling period \( T \) to obtain the discrete-time signal 
\[ x[n] = \frac{\sin\left(\frac{\pi n}{T}\right)}{\frac{\pi n}{T}} \]

i. Determine a choice for \( T \) consistent with this information.
ii. Is your choice for \( T \) in part (i) is unique? If so, explain why. If not, specify another choice of \( T \) consistent with the information given.
3. For the system shown in Fig. 2, $X(e^{j\omega})$, the Fourier Transform of the input signal $x[n]$ is shown in Fig. 3.

For each of the following choices of $L$ and $M$, specify the maximum possible value of $\omega_0$ such that $\tilde{X}_d(e^{j\omega}) = aX(e^{j\frac{M\omega}{L}})$ for some constant $a$.

(a) $M=3$, $L=2$
(b) $M=2$, $L=3$
4. Fig. 4 shows a continuous-time filter that is implemented using an LTI discrete-time filter ideal low pass filter with frequency response over $-\pi \leq \omega \leq \pi$ as

$$H(e^{j\omega}) = \begin{cases} 
1 & |\omega| < \omega_c, \\
0 & \omega_c < |\omega| \leq \pi
\end{cases}$$

![Figure 4: system for problem 4](image)

(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Fig. 5 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega}), Y(e^{j\omega}), Y_c(j\Omega)$ for $\frac{1}{T} = 2 \times 10^4$

![Figure 5: $X(e^{j\omega})$, the Fourier Transform of the input signal $x[n]$](image)

(b) For $\frac{1}{T} = 6 \times 10^3$ and for input signals $x_c(t)$, whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency $\omega_c$ of the filter $H(e^{j\omega})$ for which no aliasing occurs. For the maximum choice of $\omega_c$, specify $H_c(j\Omega)$

5. Consider the discrete-time system shown in Fig. 6

![Figure 6: discrete-time system in problem 5](image)

where

i. L is an integer

ii. $x_c[n] = \begin{cases} x[n] & n = kL, \; k \text{ is integer}, \\
0 & \text{otherwise}
\end{cases}$
iii $y[n] = y[nL]$

iv $H(e^{j\omega}) = \begin{cases} L & |\omega| < \frac{\pi}{4}, \\ 0 & \frac{\pi}{4} < |\omega| \leq \pi \end{cases}$

(a) Assume that $L=2$ and that $X(e^{j\omega})$, the DTFT of $x[n]$, is real and is shown in Fig. 7. Make an appropriately labeled sketch of $X_e(e^{j\omega}), Y(e^{j\omega}), Y_e(e^{j\omega})$. Be sure to clearly label amplitudes and frequencies.

(b) For $L=2$, the overall system is LTI. Determine and sketch the magnitude of the frequency response of the overall system $|H_{eff}(e^{j\omega})|$. 

(c) For $L=6$, the overall system is still LTI. Determine and sketch the magnitude of the frequency response of the overall system $|H_{eff}(e^{j\omega})|$. 

6. For the system shown in Fig. 8 find an expression for $y[n]$ in terms of $x[n]$. Simplify the expression as much as possible. Each of the three blocks (upsampling, interp., and downsampling) correspond to the system in Fig. 2 and the interpolator is the lowpass filter in Fig. 2.

Figure 7: $X(e^{j\omega})$, the Fourier Transform of the input signal $x[n]$

Figure 8: Discrete-time system for problem 6