1. Suppose that we wish to design an FIR lowpass filter with the following specifications:

\[ 0.92 < H(e^{j\omega}) < 1.02 \quad 0 \leq |\omega| \leq 0.63\pi, \]

\[ |H(e^{j\omega})| < 0.1 \quad 0.65\pi \leq |\omega| \leq \pi \]

by applying a window to the impulse response \( h_d[n] \) for the ideal discrete-time lowpass filter with cutoff \( \omega_c = 0.64\pi \).

(a) For each of the following windows: Hamming, Hanning, and Bartlett modify the MATLAB code 'hw8_prob1.m' to determine the minimum value of \( M \) that satisfies the aforementioned specification.

(b) To support your answer, for each window plot the frequency response of the filter you generated in part (a). Show that with \( M - 1 \) the constraints are not satisfied.

2. Consider a Type I Chebyshev lowpass filter with the following 4th order system function

\[ H_{lp}(Z) = \frac{0.0042(1 + Z^{-1})^4}{(1 - 1.4424Z^{-1} + 0.5851Z^{-2})(1 - 1.2973Z^{-1} + 0.8229Z^{-2})} \]

to meet the specifications

\[ 0.95 \leq |H_{lp}(e^{j\theta})| \leq 1.05 \quad 0 \leq |\theta| \leq 0.25\pi \]

\[ |H_{lp}(e^{j\theta})| \leq 0.1 \quad 0.35\pi \leq |\theta| \leq \pi \]

The frequency response of which is shown in the figure

(a) Transform this filter into a highpass filter with passband cutoff frequency \( \omega_p = 0.6\pi \) using

\[ H(z) = H_{lp}(Z)|_{Z^{-1} \rightarrow z^{-1-\alpha^{-1}}} \]

and \( \alpha = -\frac{\cos(\frac{\theta_p + \omega_p}{2})}{\cos(\frac{|\theta_p - \omega_p|}{2})} \)
(b) Plot the magnitude response of the new highpass filter.

3. Consider a continuous-time low-pass filter $H_c(s)$ with passband and stopband specifications

$$1 - \delta_1 \leq |H_c(j\Omega)| \leq 1 + \delta_1 \quad \Omega \leq \Omega_p$$

$$|H_c(j\Omega)| \leq \delta_2 \quad \Omega_s \leq |\Omega|$$

This filter is transformed to a low-pass discrete-time filter $H_1(z)$ by the transformation

$$H_1(z) = H_c(s)\mid_{s = \frac{1-z^{-1}}{1+z^{-1}}}$$

and the same continuous-time filter is transformed to a high-pass discrete-time filter by the transformation

$$H_2(z) = H_c(s)\mid_{s = \frac{1+z^{-1}}{1-z^{-1}}}$$

(a) Determine a relationship between the passband cutoff frequency $|\Omega_p|$ of the continuous-time low-pass filter and the passband cutoff frequency $\omega_{p1}$ of the discrete-time low-pass filter.

(b) Determine a relationship between the passband cutoff frequency $|\Omega_p|$ of the continuous-time low-pass filter and the passband cutoff frequency $\omega_{p2}$ of the discrete-time high-pass filter.

(c) Determine a relationship between the passband cutoff frequency $\omega_{p1}$ of the discrete-time low-pass filter and the passband cutoff frequency $\omega_{p2}$ of the discrete-time high-pass filter.

(d) The network in the figure depicts an implementation of the discrete-time low-pass filter with system function $H_1(z)$. The coefficients $A$, $B$, $C$ and $D$ are real. How should these coefficients be modified to obtain a network that implements the discrete-time high-pass filter with system function $H_2(z)$?

4. Let the $h_d[n]$ denote the impulse response of an ideal desired system with corresponding frequency response $H_d(e^{j\omega})$, and let $h[n]$ and $H(e^{j\omega})$ denote the impulse response and frequency response, respectively, of an FIR approximation to the ideal system. Assume that $h[n] = 0$ for $n < 0$ and $n > M$. We wish to choose the $(M + 1)$ samples of the impulse response so as to minimize the mean-squared error of the frequency response defined as

$$e^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H_d(e^{j\omega}) - H(e^{j\omega})|^2 d\omega$$

(a) Use Parseval’s relation to express the error function in terms of the sequences $h_d[n]$ and $h[n]$

(b) Using the result of part (a), determine the values of $h[n]$ for $0 \leq n \leq M$ that minimizes $e^2$

(c) The FIR filter determined in part (b) could have been obtained by a windowing operation. That is, $h[n]$ could have been obtained by multiplying the desired infinite-length sequence $h_d[n]$ by a certain finite-length sequence $w[n]$. Determine the necessary window $w[n]$ such that the optimal impulse response is $h[n] = w[n]h_d[n]$. 
