CS 321 Activity 2

1. Consider the CNF grammar \( G = (V,T,S,P) \) where

\[
V = \{ S, A, B, C, D \}, \quad T = \{ a, b, c \}, \quad S = S \text{ and } P \text{ is given below.}
\]

\[
\begin{align*}
S & \rightarrow AB | AC \\
A & \rightarrow AC | AB | a \\
B & \rightarrow BB | BC | b \\
C & \rightarrow AC | CC | c | b
\end{align*}
\]

Use the CKY to determine if the strings \( w_1 = acbb \) and \( w_2 = bbca \) are in the language \( L(G) \). If the string is in \( L(G) \) construct the parse tree.
CS 321 Activity 2

2. Write a context-free grammar for the following language.

\[ L = \{ a^m b^{m^2 n} : m \geq 0, n > 0 \} \cup \{ b^n a^n : n > 0 \} \]
3. Consider $L = \{ a^p b^{2n} c^p \mid p < n, \ n > 0 \}$. Prove $L$ is not a context-free language.
4. Consider \( L = \{ a^n b^p c^q \mid n + q = p, p > 0, q > 0, n > 0, \text{ and } n \text{ is even} \} \), \( \Sigma = \{a, b, c\} \).

(a) List four strings in \( L \).

(b) Verbally describe and give the formal definition of an NPDA \( M \) that accepts \( L \) by final state. Assume \( Z \) is on the top of the stack when \( M \) starts.

You can use a transition graph to represent the transition function. Identify the start state by an arrow and final states by double circles. The format of the labels on the edges should be: \( a, b; x \) where \( a \) is an input character, \( b \) is the symbol popped off the top of the stack and \( x \) is the symbol(s) pushed onto the stack.
5. Convert the following CFG to CNF

\[
S \rightarrow ABa \mid AC \\
A \rightarrow Ab \mid a \\
B \rightarrow b \mid C \mid \lambda \\
C \rightarrow aa \mid AA
\]

6. Consider the following languages. Write “REG” if it is regular, “CFL:” if it is a CFL and not regular, and write “NOT” if it is not a CFL.

(a) \(L = \{a^nb^{2m}c^{3n} \mid n > 0\}\) \(\text{REG}\)

(b) \(L = \{a^nb^{2m} \mid m, n > 0\}\) \(\text{CFL}\)

(c) \(L = \{b^ma^nc^p \mid m > n+p, n > 0, p > 0\}\) \(\text{CFL}\)

(d) \(L = \{b^pc^n \mid n > 2p, n > 0, 0 < p < 10\}\) \(\text{CFL}\)

(e) \(L = \{w \in \{a, b, c\}^* \mid n_a(w) > n_b(w)\}\) \(\text{NOT}\)
7. Consider the context-free grammar

\[
S \rightarrow aS \mid aaC \mid aB \\
B \rightarrow b \mid ab \\
C \rightarrow aB \mid b
\]

(a) Show that the grammar is ambiguous

(b) Find an equivalent unambiguous grammar.