1. The following continuous-time input signals \( x_c(t) \) and corresponding discrete-time output signals \( x[n] \) are those of an ideal C/D as shown in Figure 1:

\[
\text{Figure 1}
\]

Specify a choice for the sampling period \( T \) that is consistent with each pair of \( x_c(t) \) and \( x[n] \). In addition, indicate whether your choice of \( T \) is unique. If not, specify all possible choices of consistent with the information given.

(a) \( x_c(t) = \sin(10\pi t), x[n] = \sin\left(\frac{\pi n}{4}\right) \)

(b) \( x_c(t) = \frac{\sin(10\pi t)}{10\pi t}, x[n] = \frac{\sin(\pi n/2)}{\pi n/2} \)

2. Your goal is using the digital time processing of analog signal system in figure 2 to implement a derivative, i.e. \( y_c(t) = \frac{d}{dt} x_c(t) \).

\[
\text{Figure 2}
\]

(a) Write \( H(j\Omega) \) for the derivative.

(b) Find \( H(e^{j\omega}) \).

(c) Find and plot \( h[n] \).

3. Figure 4 shows a continuous-time filter that is implemented using an LTI discrete-time filter ideal low pass filter with frequency response over \( -\pi < \omega < \pi \) as

\[
H(e^{j\omega}) = \begin{cases} 
1 & |\omega| < \omega_c \\
0 & \omega_c < |\omega| \leq \pi 
\end{cases}
\]
(a) If the continuous-time Fourier transform of \( x_c(t) \), namely \( X_c(j\Omega) \), is shown in Figure 5 and \( \omega_c = \frac{\pi}{5} \), sketch and label \( X(e^{j\omega}) \), \( Y(e^{j\omega}) \), \( Y_c(j\Omega) \) for \( \frac{1}{T} = 2 \times 10^4 \).

(b) For \( \frac{1}{T} = 6 \times 10^3 \) and for input signals \( x_c(t) \), whose spectra are bandlimited to \( |\Omega| < 2\pi \times 5 \times 10^3 \) (but otherwise unconstrained), what is the maximum choice of the cutoff frequency \( \omega_c \) of the filter \( H(e^{j\omega}) \) for which no aliasing occurs. For the maximum choice of \( \omega_c \), specify \( H(j\Omega) \).

4. Each of the following parts lists an input signal \( x[n] \) and the upsampling and downsampling rates \( L \) and \( M \) for the system in Figure 6. Determine the corresponding output \( x_d[n] \).
(a) \( x[n] = \sin\left(\frac{2\pi n}{3}\right) / \pi n, \ L = 4, \ M = 3 \)

(b) \( x[n] = \sin\left(\frac{3\pi n}{4}\right), \ L = 3, \ M = 5 \)

5. For the system shown in Figure 7:

Find an expression for \( y[n] \) in terms of \( x[n] \). Simplify the expression as much as possible.