1. Consider a causal continuous-time system with impulse response $h_c(t)$ and system function

$$H_c(s) = \frac{s + a}{(s + a)^2 + b^2}$$

a) Use impulse invariance to determine $H_1(z)$ for discrete-time system such that

$$h_1[n] = T h_c(nT).$$

b) Plot the magnitude and frequency response for both the continuous-time and discrete-time filters using MATLAB assuming $a = 1$ and $b = 1$. (Hint: Use MATLAB commands `freqs` and `freqz` to get the frequency responses).

2. A discrete-time low-pass filter is to be designed by applying the impulse invariance method to a continuous time Butterworth filter having magnitude-squared function

$$|H(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_k}\right)^{2N}}$$

The specifications for the discrete-time system are:

$$0.9 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq |\omega| \leq 0.2\pi$$

$$|H(e^{j\omega})| \leq 0.1, \quad 0.3\pi \leq |\omega| \leq \pi$$

Assume that aliasing will not be a problem; i.e., design the continuous-time Butterworth filter to meet passband and stopband specification as determined by the desired discrete-time filter.

a) Sketch the tolerance bounds on the magnitude of the frequency response, $|H_c(j\Omega)|$, of the continuous time Butterworth filter such that after application of the impulse invariance method (i.e., $h[n] = T_d h_c(nT_d)$), the resulting discrete-time filter will satisfy the given design specifications. Assume that $T_d = 1$.

b) Determine the integer order $N$ and the quantity $\Omega_k$ such that the continuous-time Butterworth filter exactly meets the specifications determined in part (a) at the passband edge.

c) Determine system function $H(s)$, and get $H(z)$ by impulse invariance.

d) Use MATLAB to plot the magnitude and phase response of $H(z)$. 
3. We wish to use impulse invariance or the bilinear transformation to design a discrete-time filter that meets specifications of the following form:

\[
1 - \delta_i \leq |H(e^{j\omega})| \leq 1 + \delta_i, \quad 0 \leq |\omega| \leq \omega_p
\]

\[
|H(e^{j\omega})| \leq \delta_2, \quad \omega_1 \leq |\omega| \leq \pi
\]

For historical reasons, most of the design formulas, tables, or charts for continuous-time filters are normally specified with a peak gain of unity in the passband; i.e.,

\[
1 - \delta_i' \leq |H_c(j\Omega)| \leq 1, \quad 0 \leq |\Omega| \leq \Omega_p
\]

\[
|H_c(j\Omega)| \leq \delta_2', \quad \Omega_1 \leq |\Omega| \leq \Omega
\]

(2)

Useful design charts for continuous-time filters specified in this form were given by Rabiner, Kaiser, Herrmann, and Dolan.

a) To use tables and charts to design discrete-time systems with peak gain of \((1 + \delta_i')\), it is necessary to convert the discrete-time specifications into specifications of the form of (2). This can be done by dividing the discrete-time specifications by \((1 + \delta_i')\).

b) Suppose we know a discrete-time filter:

\[
0.9 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq |\omega| \leq 0.3\pi
\]

\[
|H(e^{j\omega})| \leq 0.1, \quad 0.4\pi \leq |\omega| \leq \pi
\]

This filter can be converted to a filter satisfying a set of specifications such as those in (1) by multiplying a constant of the form \((1 + \delta_i')\). Find the required value of \(\delta_i\) and the corresponding value of \(\delta_2\) for this filter, and use \(H(z)\) to determine the coefficient of the system function of the new filter. (You do not need to write down the detail of \(H(z)\), just use \(H(z)\)).

4. The system function of a discrete-time system is

\[
H(z) = \frac{2}{1 - e^{-0.2}z^{-1}} - \frac{1}{1 - e^{-0.4}z^{-1}}
\]

a) Assume that this discrete-time filter was designed by the impulse invariance method with \(T_d = 2\); i.e. \(h[n] = 2h_c(2n)\), where \(h_c(t)\) is real. Find the system function \(H_c(s)\) of a continuous-time filter that could have been the basis for the design.

b) Assume that \(H(z)\) was obtained by the bilinear transform method with \(T_d = 2\). Find the system function \(H_c(s)\) that could have been the basis for the design.