Suppose that we wish to design an FIR low pass filter with the following specifications:

\[ 0.92 < H \left( e^{j\omega} \right) < 1.02 \quad 0 \leq |\omega| \leq 0.63\pi \]

\[ |H \left( e^{j\omega} \right)| < 0.1 \quad 0.65\pi \leq |\omega| \leq \pi \]

By applying a window to the impulse response \( h_d[n] \) for the ideal discrete-time low pass filter with cutoff \( \omega_c = 0.64\pi \).

(a) For each the following windows: Hamming, Hanning, and Bartlett modify the MATLAB code ‘hw8-prob1.m’ to determine the minimum value of \( M \) that satisfies the aforementioned specification.

(b) To support your answer, for each window plot the frequency response of the filter you generated in part (a), also show that constrains are not satisfied with \( M-1 \).

2. Consider a continuous-time system with system function

\[ H_c(s) = \frac{1}{s} \]

This system is called an integrator, since the output \( y_c(t) \) is related to the input \( x_c(t) \) by

\[ y_c(t) = \int_{-\infty}^{t} x_c(\tau) d\tau \]

Suppose a discrete-time system is obtained by applying the bilinear transformation to \( H_c(s) \).

(a) What is the system function \( H(z) \) of the resulting discrete-time system? What is the impulse response \( h[n] \)?

(b) Write the difference equation. Is it stable?

(c) Obtain an expression for the frequency response \( H \left( e^{j\omega} \right) \) of the system. Sketch the magnitude and phase of \( H \left( e^{j\omega} \right) \) by MATLAB.

3. Consider designing a discrete-time filter with system function \( H(z) \) from a continuous-time filter with rational system function \( H_c(s) \) by the transformation

\[ H(z) = H_c(s)|_{s = \frac{\beta}{\alpha(1-\zeta^{-\alpha})(1+\zeta^{-\alpha})}} \]

Where \( \alpha \) is an nonzero integer and \( \beta \) is real.
(a) If \( \alpha > 0 \), for what values of \( \beta \) does a stable, causal continuous-time filter with rational \( H_c(s) \) always lead to a stable, causal discrete-time filter with rational \( H(z) \)?

(b) If \( \alpha < 0 \), for what values of \( \beta \) does a stable, causal continuous-time filter with rational \( H_c(s) \) always lead to a stable, causal discrete-time filter with rational \( H(z) \)?

4. As discussed in Chapter 12, an ideal discrete-time Hilbert transformer is a system that introduces -90 degrees (-\( \pi / 2 \) radians) of phase shift for \( 0 < \omega < \pi \) and +90 degrees (\( \pi / 2 \) radians) of phase shift for \( -\pi < \omega < 0 \). The magnitude of the frequency response is constant (unity) for \( 0 < \omega < \pi \) and for \( -\pi < \omega < 0 \). Such systems are also called ideal 90-degree phase shifters.

(a) Give an equation for the ideal desired frequency response \( H_d(e^{j\omega}) \) of an ideal discrete-time Hilbert transformer that also includes constant (nonzero) group delay. Plot the phase response of this system for \( -\pi < \omega < \pi \).

(b) Suppose that we wish to use the window method to design a linear-phase approximation to the ideal Hilbert transformer. Use \( H_d(e^{j\omega}) \) given in part (a) to determine the ideal impulse response \( h[n] \) if the FIR system is to be such that \( h[n] = 0 \) for \( n < 0 \) and \( n > M \).

(c) What type(s) of FIR linear-phase systems (I, II, III, or IV) can be used to approximate the ideal Hilbert transformer in part (a)?

(d) What is the delay of the system if \( M = 30 \)? Sketch the magnitude of the frequency response of the FIR approximation for this case by MATLAB, assuming a rectangular window.

(e) What is the delay of the system if \( M = 31 \)? Sketch the magnitude of the frequency response of the FIR approximation for this case by MATLAB, assuming a rectangular window.

5. (Bonus) Download the attached file and load it into your MATLAB by using ‘HW8 Bonus.mat’.

A piece of music is added with a high-pass noise. Please design a low-pass filter to eliminate this noise. The specification of that high-pass noise is:

\[ \text{stop frequency} = 10kHz \]
\[ \text{pass frequency} = 12kHz \]

The original music has a sample rate equals to \( f_{sample} = 44.1kHz \).

You can use command ‘sound’ in MATLAB to play the music. [\( \text{sound(y, s)} \) sends audio signal y to the speaker at sample rate \( s \).]

For the filtering, please use command \( y = \text{filter}(b,a,x) \). [\( y = \text{filter}(b,a,x) \) filters the input data x, using a rational transfer function defined by the numerator and denominator coefficients b and a, respectively.] Hint: your low pass filter only need to cover the general frequency range of
sound. Choose any filter of your choice to filter the signal.  
(a) Please attach the MATLAB code for the problem with the information about the choice of filter, cutoff frequency and the order of the pole.  
(b) Plot the magnitude and frequency response for the filter of your choice in MATLAB.