1. Consider the sequence \( x[n] \) given by \( x[n] = \alpha^n u[n] \). Assume \( |\alpha| < 1 \). A periodic sequence \( x[n] \) is constructed from \( x[n] \) in the following way:

\[
 x[n] = \sum_{r=-\infty}^{\infty} x[n + rN]
\]

(a) Determine the Fourier transform \( X(e^{j\omega}) \) of \( x[n] \).

(b) Determine the DFS coefficients \( x[k] \) of \( x[n] \).

(c) How is \( x[k] \) related to \( X(e^{j\omega}) \).

2. Compute the DFS of each of the following finite-length sequences considered to be of length \( N \) (where \( N \) is even):

(a) \( x[n] = \delta[n] \)

(b) \( x[n] = \delta[n-n_0] \), \( 0 \leq n_0 \leq N-1 \)

(c) \( x[n] = \begin{cases} 1, & n \text{ even}, \ 0 \leq n \leq N-1 \\ 0, & n \text{ odd}, \ 0 \leq n \leq N-1 \end{cases} \)

(d) \( x[n] = \begin{cases} 1, & 0 \leq n \leq N/2 - 1 \\ 0, & N/2 \leq n \leq N-1 \end{cases} \)

(e) \( x[n] = \begin{cases} \alpha^n, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases} \)

3. Consider the finite-length sequence \( x[n] \) in Figure 1. The five-point DFT of \( x[n] \) is denoted by \( x[k] \). Plot the sequence \( y[n] \) whose DFT is

\[
 Y[k] = W_5^{-2k} X[k]
\]

4. Consider the six-point sequence

\[
 x[n] = 6\delta[n] + 5\delta[n-1] + 4\delta[n-2] + 3\delta[n-3] + 2\delta[n-4] + \delta[n-5]
\]
Show in the Figure 2.

(a) Compute the DTFT of $x[n]$.

(b) Compute the DFT coefficients for $x[n]$, i.e., $X[k]$ for $k = 0, 1, 2, 3, 4, 5$

5. The two eight-point sequences $x_1[n]$ and $x_2[n]$ shown in Figure 3 have DFTs $X_1[k]$ and $X_2[k]$, respectively. Determine the relationship between $X_1[k]$ and $X_2[k]$. 

Figure 2

Figure 3