1. The continuous-time signal
   \( x_c(t) = \sin (20\pi t) + \cos (40\pi t) \)

   is sampled with a sampling period \( T \) to obtain the discrete-time signal
   \( x[n] = \sin \left( \frac{n\pi}{5} \right) + \cos \left( \frac{2\pi n}{5} \right) \)

   (a) Determine a choice for \( T \) consistent with this information.
   (b) Is your choice for \( T \) in part (a) unique? If so, explain why. If not, specify another choice of \( T \) consistent with the information given.

2. The continuous-time signal
   \( x_c(t) = \frac{\sin (10\pi t)}{10\pi t} \)

   is sampled with a sampling period \( T \) to obtain the discrete-time signal
   \( x[n] = \frac{\sin \left( \frac{n\pi}{2} \right)}{\left( \frac{n\pi}{2} \right)} \)

   (a) Determine a choice for \( T \) consistent with this information.
   (b) Is your choice for \( T \) in part (a) unique? If so, explain why. If not, specify another choice of \( T \) consistent with the information given.

3. Use the system shown in Fig. 1. below to implement a differentiator:

   \( y_c(t) = \frac{d}{dt} x_c(t) \)

   (C/D: An ideal continuous-to-discrete time converter, D/C: An ideal discrete-to-continuous time converter)

   \( H_c(j\Omega) \)

   \[ \begin{array}{c}
   x_c(t) \text{ (input)} \\
   C/D \text{ (converter)} \\
   x_d[n] \text{ (sampled)} \\
   H_c(e^{j\omega T}) \text{ (filter)} \\
   y_d[n] \text{ (output)} \\
   D/C \text{ (converter)} \\
   y_c(t) \text{ (reconstructed)} \\
   \end{array} \]

   Fig. 1. For Problem 3

   a. Write \( H_c(j\Omega) \) for the derivative
   b. Find \( H_c(e^{j\omega T}) \)
   c. Find and plot \( h[n] \)
4. Each of the following parts lists an input signal $x[n]$ and the Up-sampling and Down-sampling rates $L$ and $M$ for the system in Fig. 2. Determine the corresponding output $\tilde{x}_d[n]$

(a) $x[n] = \sin(2\pi n/3)/\pi n$, $L = 4$, $M = 3$
(b) $x[n] = \sin(3\pi n/4)$, $L = 6$, $M = 7$

5. For the system shown in Fig. 2, $X(e^{j\omega})$, the Fourier transform of the input signal $x[n]$, is shown in Fig. 3.

For each of the following choices of $L$ and $M$, specify the maximum possible value of $\omega_0$ such that $\tilde{X}_d(e^{j\omega}) = aX(e^{jM\omega/L})$ for some constant $a$.

(a) $M = 3$, $L = 2$
(b) $M = 2$, $L = 3$

6. Fig. 4 shows a continuous-time filter that is implemented using an LTI discrete-time filter ideal lowpass filter with frequency response over $-\pi \leq \omega \leq \pi$ as
(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Fig. 5. and $\omega_c = \pi/5$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$, and $Y_c(j\Omega)$ for $1/T = 2 \times 10^4$

(b) For $1/T = 6 \times 10^3$ and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency $\omega_c$ of the filter $H(e^{j\omega})$ for which no aliasing occurs. For this maximum choice of $\omega_c$, specify $H_c(j\Omega)$. 

Fig. 4. Continuous-Time filter using a discrete-time LPF

Fig. 5. Continuous-Time Fourier Transform of $x_c(t)$