1. Suppose $x_c(t)$ is a periodic continuous-time signal with period 1 ms and for which the Fourier series is:

$$x_c(t) = \sum_{k=-9}^{9} a_k e^{j(2000\pi kt)}$$

The Fourier series coefficients $a_k$ are zero for $|k| > 9$. $x_c(t)$ is sampled with a sample spacing $T = \frac{1}{6} \times 10^{-3}$ s to form $x[n]$. That is,

$$x[n] = x_c \left( \frac{n}{6000} \right)$$

a) Is $x[n]$ periodic and, if so, with what period?

b) Is the sampling rate above the Nyquist rate? That is, is $T$ sufficiently small to avoid aliasing?

c) Find the DFS coefficients of $x[n]$ in terms of $a_k$.

2. Compute the DFT of each of the following finite-length sequences considered to be of length $N$ (where $N$ is even):

   a) $x[n] = \delta[n]$

   b) $x[n] = \delta[n-n_0]$, $0 \leq n_0 \leq N-1$

   c) $x[n] = \begin{cases} 1, & \text{n even, } 0 \leq n \leq N-1 \\ 0, & \text{n odd, } 0 \leq n \leq N-1 \end{cases}$

   d) $x[n] = \begin{cases} 1, & 0 \leq n \leq N/2-1 \\ 0, & N/2 \leq n \leq N-1 \end{cases}$

   e) $x[n] = \begin{cases} a^n, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$

3. Consider the finite-length sequence $x[n]$ in Fig 1. below. The five-point DFT of $x[n]$ is denoted by $X[k]$. Plot the sequence $y[n]$ whose DFT is $Y[k] = W_5^{-2k} X[k]$.

   **Fig. 1.** Sequence $x[n]$ for prob. 3
4. Consider the six-point sequence:

\[ x[n] = 6\delta[n] + 5\delta[n - 1] + 4\delta[n - 2] + 3\delta[n - 3] + 2\delta[n - 4] + \delta[n - 5] \]

shown in Figure 2.

![Sequnce x[n] for prob. 4](image1)

**Fig. 2.** Sequence x[n] for prob. 4

a) Determine \( X[k] \), the six-point DFT of \( x[n] \). Express your answer in terms of \( W_6 = e^{-j2\pi/6} \).

b) Compute the DTFT of \( x[n] \).

5. The two eight-point sequences \( x_1[n] \) and \( x_2[n] \) are shown in Figure 3 have DFTs \( X_1[k] \) and \( X_2[k] \), respectively. Determine the relationship between \( X_1[k] \) and \( X_2[k] \).

![Sequences x_1[n] and x_2[n] for prob. 5](image2)

**Fig. 3.** Sequences \( x_1[n] \) and \( x_2[n] \) for prob. 5