

CS 161

Intro to CS I

Recursion



Odds and Ends

- Assignment 4 due Sunday
- Questions? `<cstdlib>`

`int x = atoi (s, c-str ())`

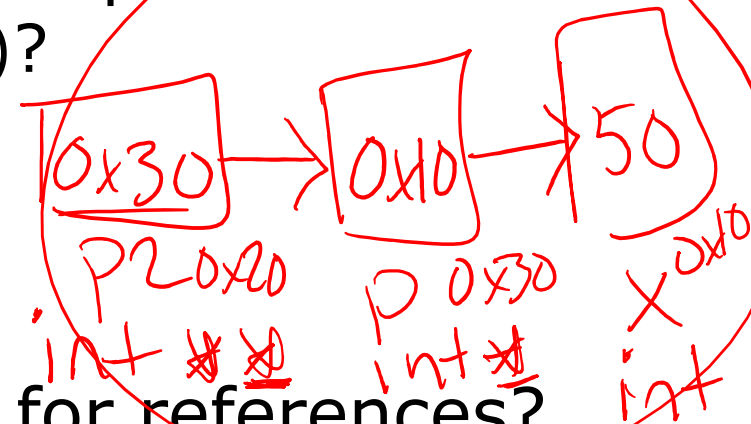
`double / float y = atof (s, c-str ())`

C-style strings

Exercise Pointers vs. References



- What if you made a pointer (p2) that points to a pointer (p) that points to an int (x)?
 - What would the picture look like?
 - Write the code for this picture.



```
int **p2, *p, x = 50;
```

- Can you make this same picture for references?
 - What if you had two references, r and r2?

```
p = &x;
r2 = &p;
```

```
cout << *p; // 50
cout << *r2; // 0x10
cout << ***r2; // 50
```

Recursion = Repetition



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- What is it?
 - Function that calls itself 1 or more times (directly or indirectly)
 - Has 1 or more base case for stopping
 - Inductive reasoning: general case must eventually be reduced to a base case



Example: Drawing Rectangles

- Iterative Solution:

```
void draw_rect(int i) {  
    for( ; i > 0; i--){  
        cout << "*****" << endl;  
        cout << " *           * " << endl;  
        cout << "*****" << endl;  
    }  
}
```

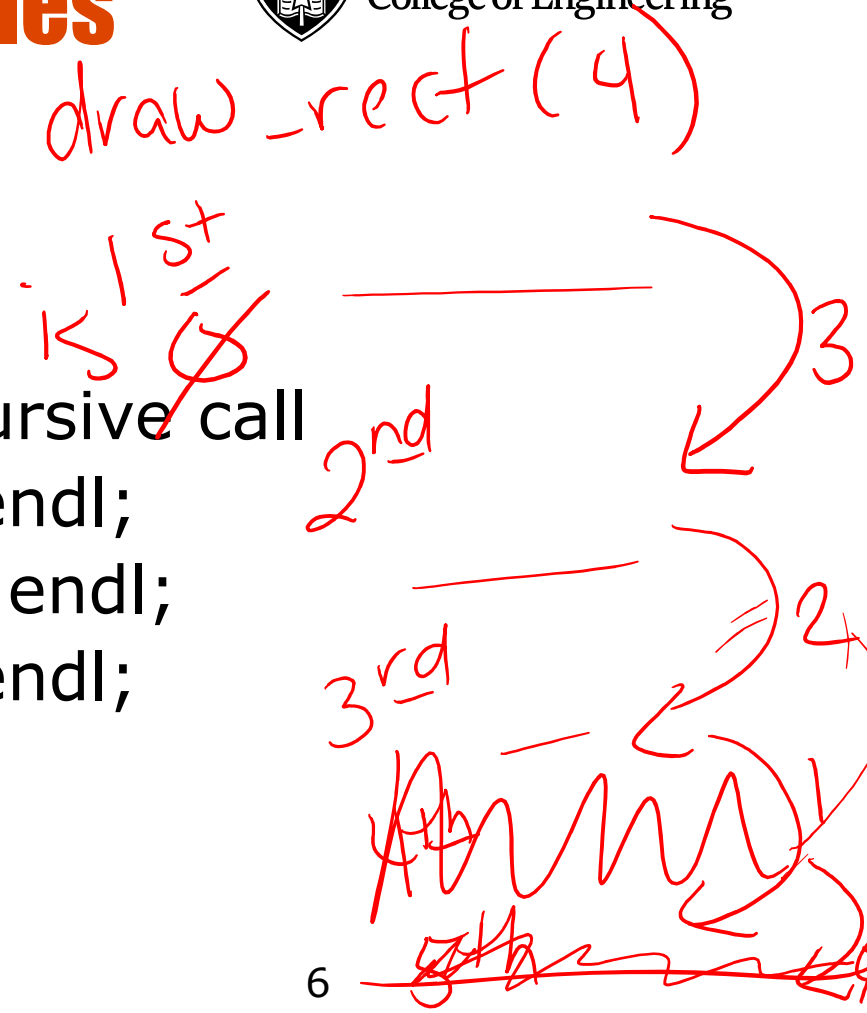
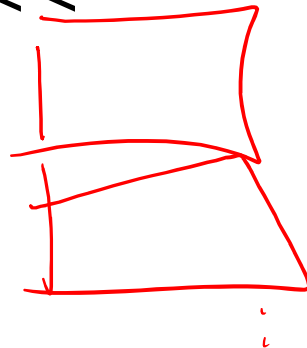
draw_rect(4);



Example: Drawing Rectangles

- Recursive Solution

```
void draw_rect(int i) {  
    if(i > 0) { //Base case  
        draw_rect(--i); //Recursive call  
        cout << "*****" << endl;  
        cout << "*          *" << endl;  
        cout << "*****" << endl;  
    }  
}
```

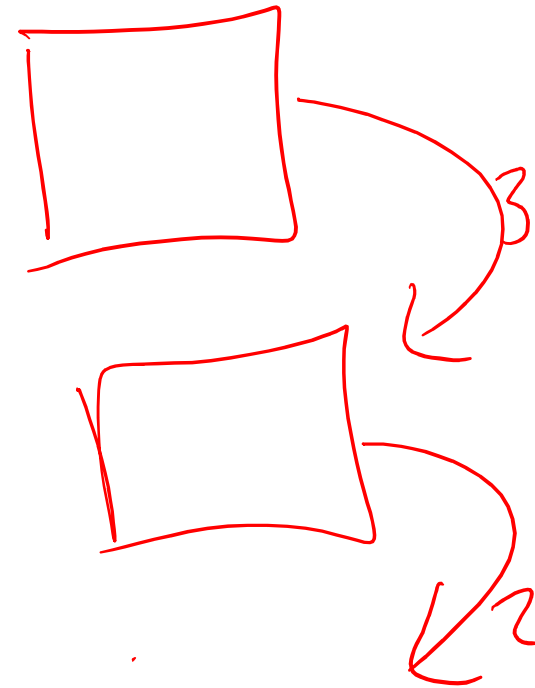


What is different when we call after?



- Recursive Solution

```
void draw_rect(int i) {  
    if(i>0) { //Base case i is 0  
        cout << "*****" << endl;  
        cout << "      *" << endl;  
        cout << "*****" << endl;  
        draw_rect(--i); //Recursive call  
    }  
}
```



Example: Factorial

- Definition

$$0! = 1;$$

$$n! = n * (n-1) * \dots * (n-(n-1)) * 1 = n * (n-1)!; n > 0$$

Iterative Factorial



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factorial(0) = 1;
factorial(n) = n*n-1*n-2*...*n-(n-1)*1;

```
long factorial(int n) {  
    long fact;  
    if(n==0)  
        fact=1;  
    else  
        for(fact=n; n > 1; n--)  
            fact=fact*(n-1);  
    return fact;  
}
```

Recursive Factorial

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

```
long factorial(int n) {  
    if (n == 0)    // Base case  
        return 1;  
    else  
        return n * factorial(n - 1);    // Recursive call  
}
```

Computing Factorial Iteratively



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factorial(4)

```
factorial(0) = 1;
```

```
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively



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$$\text{factorial}(4) = 4 * 3$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively



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$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively



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$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \\ &= 24 * 1\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively



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$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \\ &= 24 * 1 \\ &= 24\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Recursively



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factorial(4)

```
factorial(0) = 1;
```

```
factorial(n) = n*factorial(n-1);
```


Computing Factorial Recursively



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```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\text{factorial}(4) = 4 * \text{factorial}(3)$$

Computing Factorial Recursively



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```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= \underline{4} * (3 * \text{factorial}(2)) \end{aligned}$$

Computing Factorial Recursively



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```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1)))\end{aligned}$$

Computing Factorial Recursively



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```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \end{aligned}$$

Computing Factorial Recursively



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```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1)))\end{aligned}$$

Computing Factorial Recursively



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```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \end{aligned}$$

Computing Factorial Recursively



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```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \end{aligned}$$

Computing Factorial Recursively



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```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \end{aligned}$$

Computing Factorial Recursively



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`factorial(0) = 1;`

`factorial(n) = n*factorial(n-1);`

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \\ &= 24 \end{aligned}$$

Differences

- Pros
 - Readability
- Cons
 - Efficiency
 - Memory



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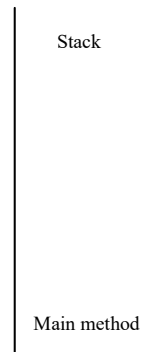
Recursive Factorial



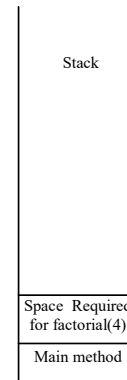
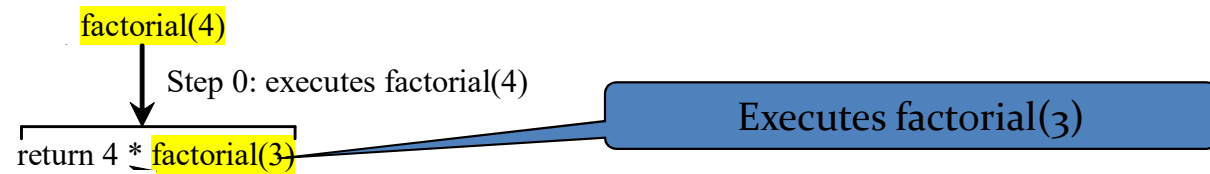
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factorial(4)

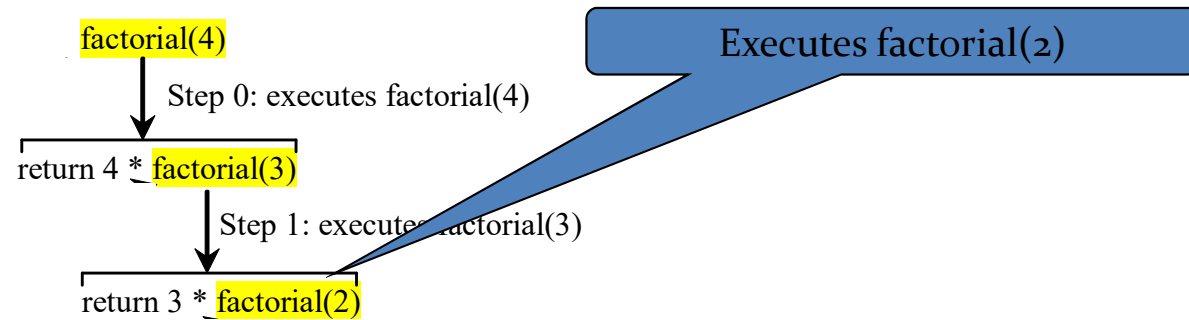
Executes factorial(4)



Recursive Factorial

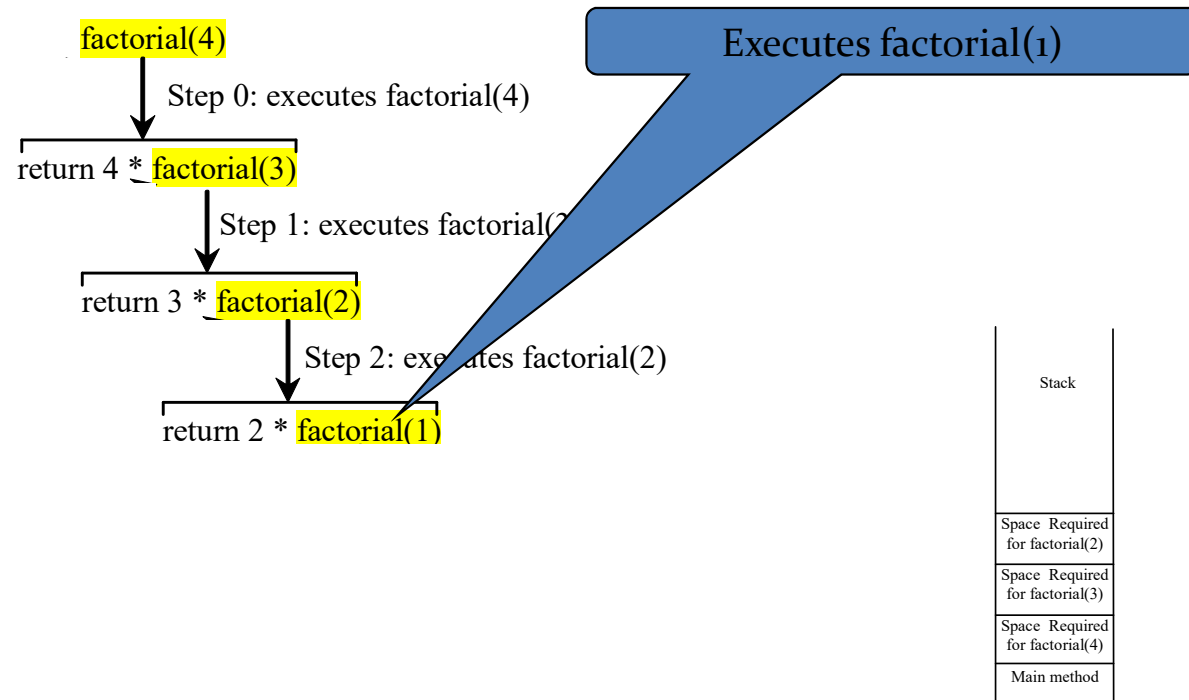


Recursive Factorial

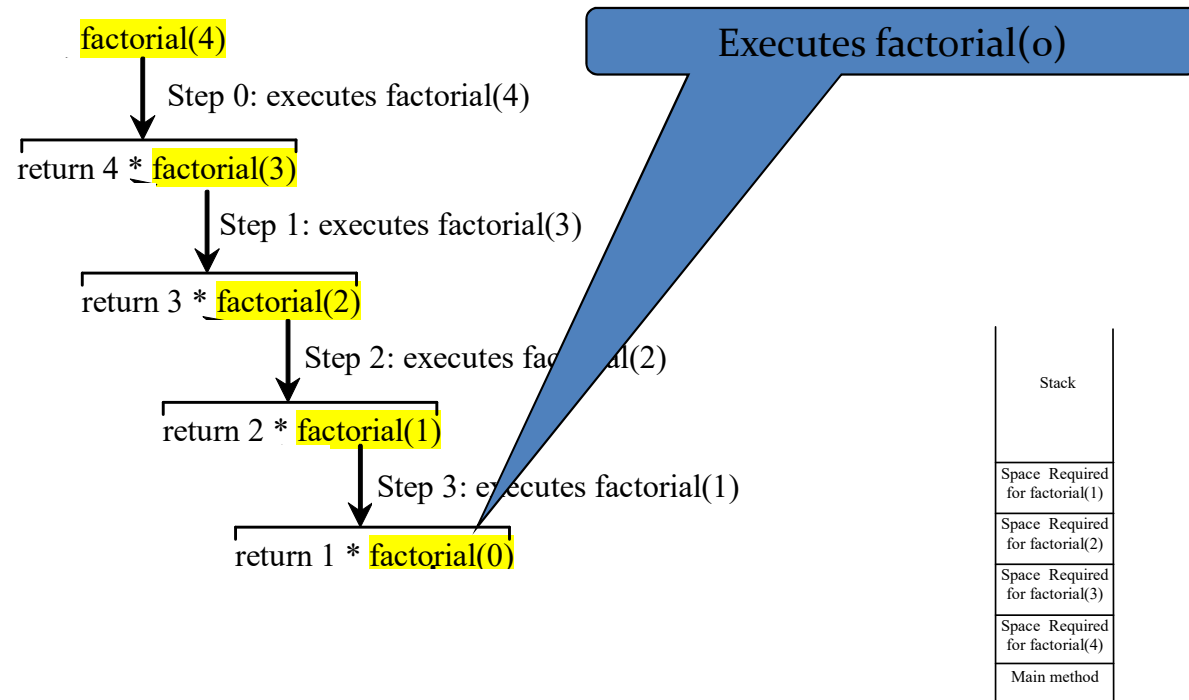


Stack
Space Required for factorial(3)
Space Required for factorial(4)
Main method

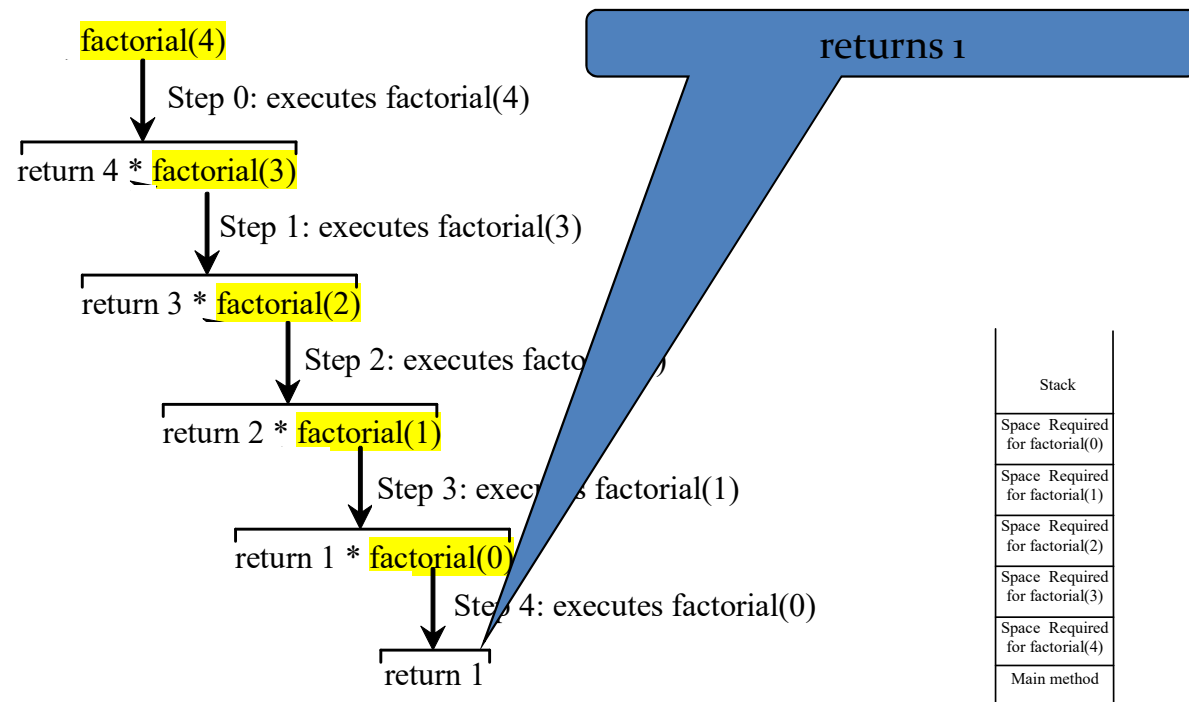
Recursive Factorial



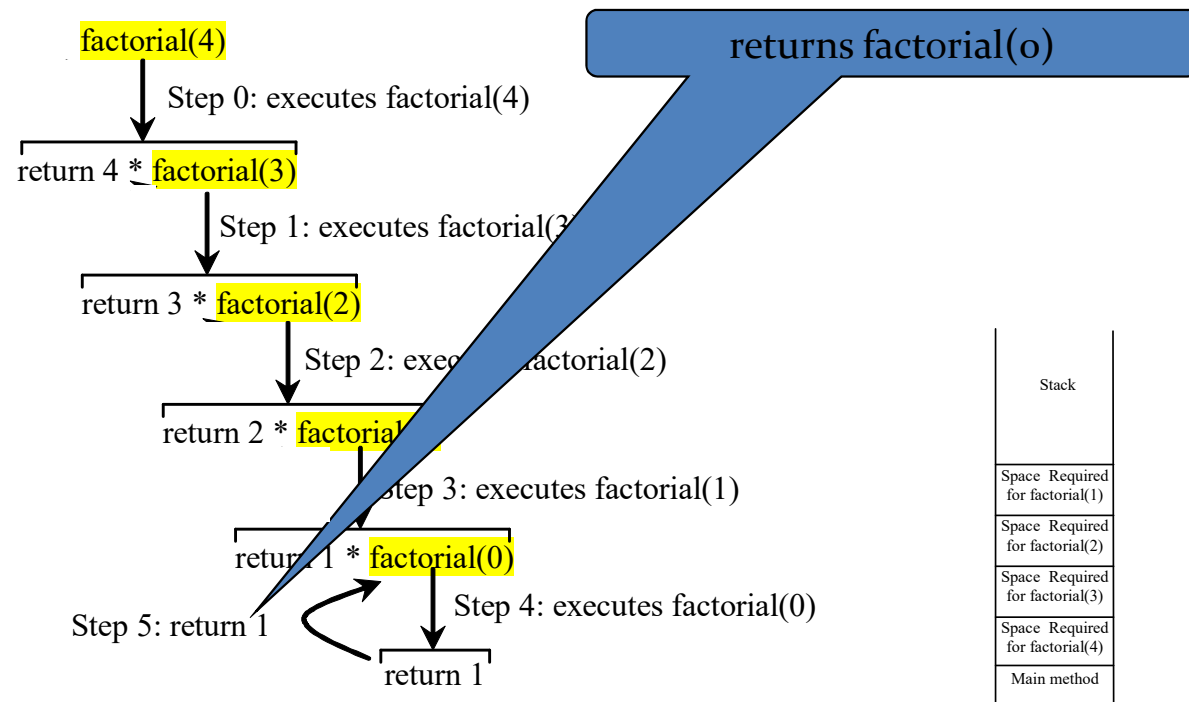
Recursive Factorial



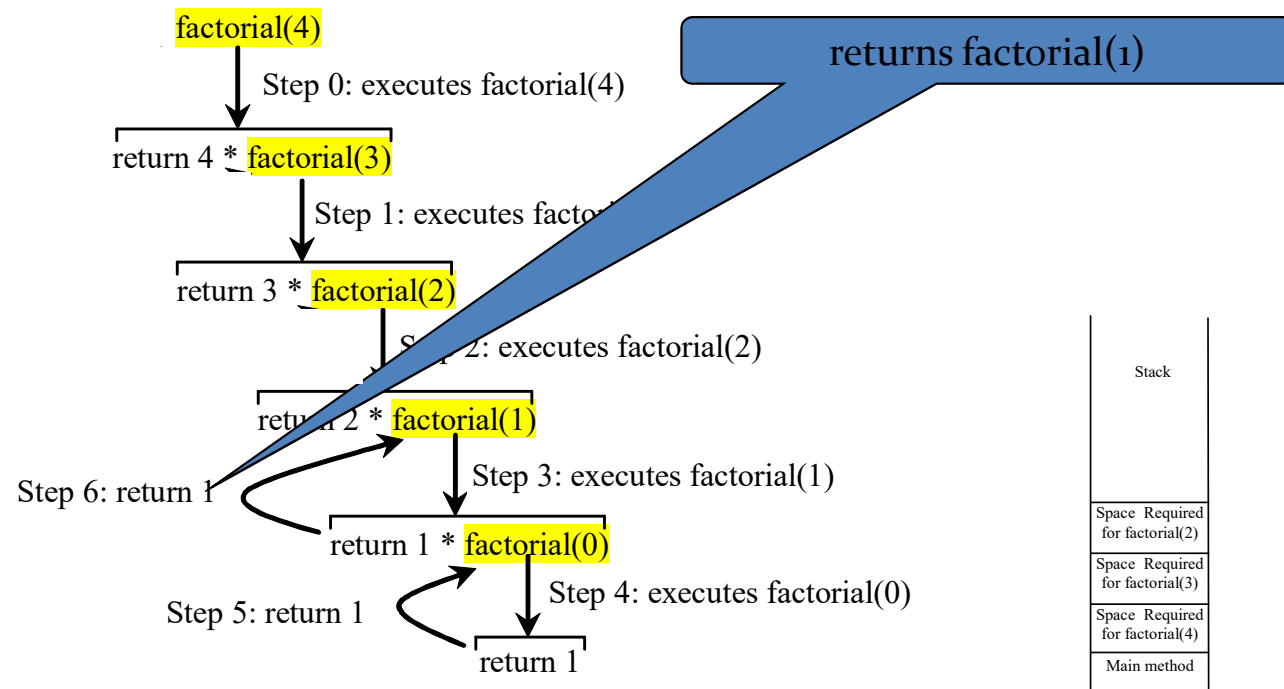
Recursive Factorial



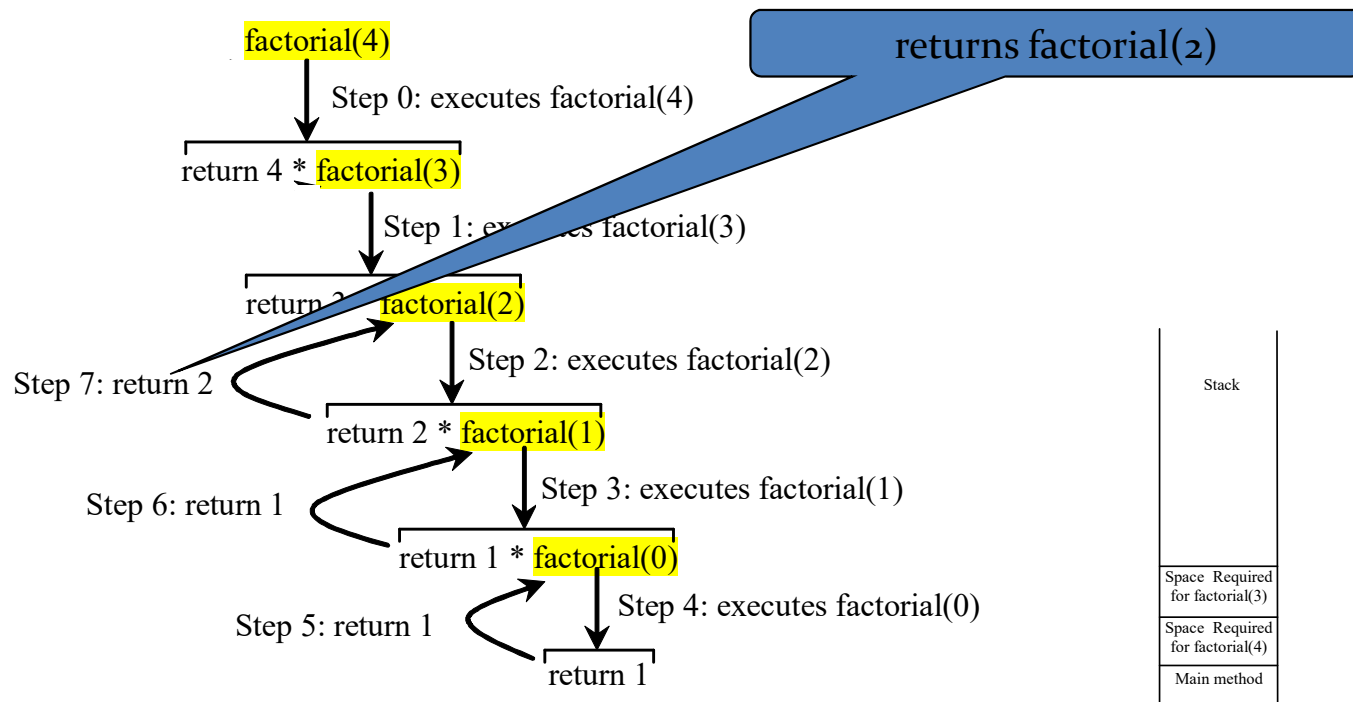
Recursive Factorial



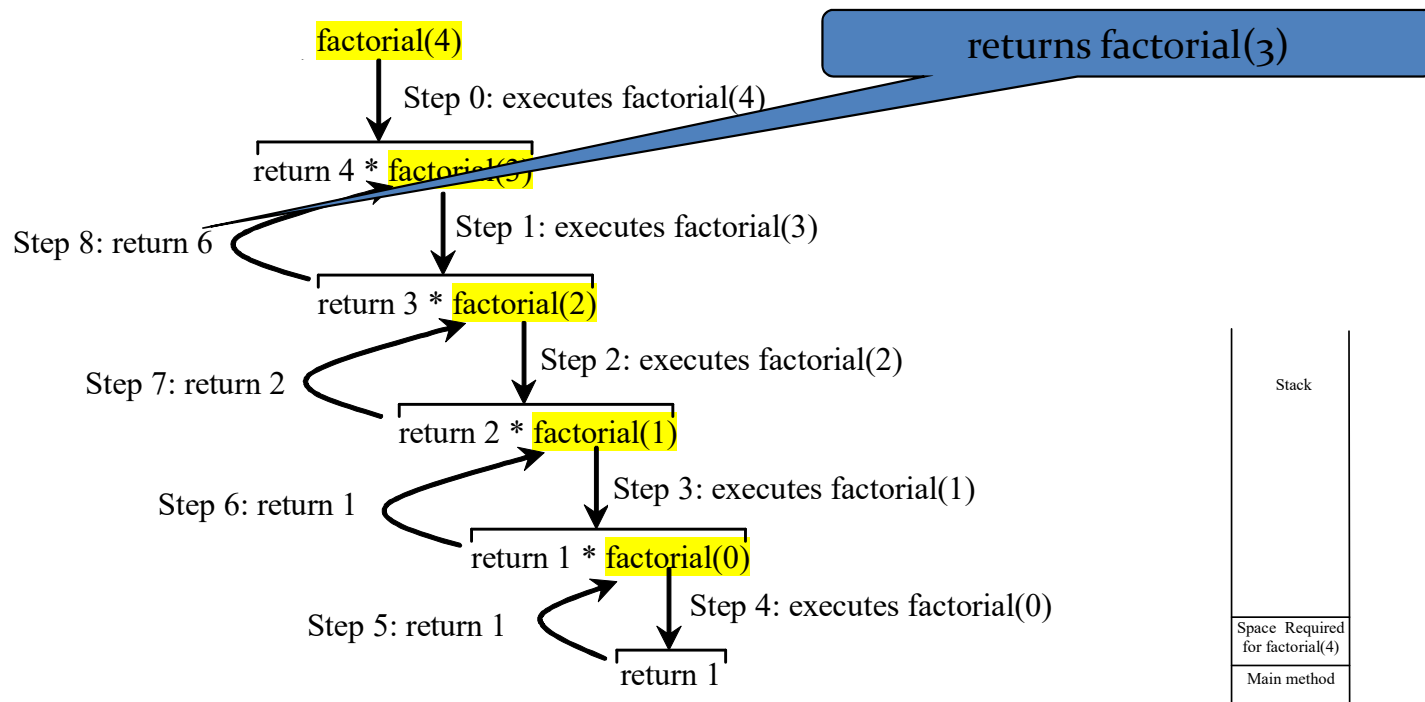
Recursive Factorial



Recursive Factorial



Recursive Factorial



Recursive Factorial

