1. (Modified version of Exercise 3.1 in the textbook) Suppose you have three Boolean random variables $X_1, X_2, X_3$. Write out a discrete joint probability distribution $P(X_1, X_2, X_3)$ where for each $i \neq j$, we have that $(X_i \perp X_j) \in I(P)$, but we also have that $((X_1, X_2) \perp X_3) \notin I(P)$. This should be a table with $2^3$ rows (one for each combination of values for the three random variables). Explain your solution. [5 points]

2. Answer true or false to the following conditional independence statements using the graph below. For partial credit, show the paths that are blocked or not blocked. [15 points]
   a) $C \perp D \mid \{B, H\}$
   b) $I \perp C$
   c) $I \perp H \mid F$
   d) $B \perp E \mid \{D, G\}$
   e) $A \perp I \mid \{E\}$
3. (Exercise 3.7 in the textbook) Show how you could efficiently compute the distribution over a variable $X_i$ given some assignment to all the other variables in the network: $P(X_i|x_1, ..., x_{i-1}, x_{i+1}, ..., x_n)$. Your procedure should not require the construction of the entire joint distribution $P(X_1, ..., X_n)$. Specify the computational complexity of your procedure using Big-O notation. [10 points]

4. (Modified version of Exercise 3.8 in the textbook) Let $B = (G, P)$ be a Bayesian network over some set of variables $V$. Let us denote the nodes observed as evidence as the set $Z$, and let $X$ be all of the ancestors of the nodes in $Z$ (i.e. the parents of $Z$ as well as their ancestors). Let $W = V - X - Z$ (i.e. the remaining nodes not in $X \cup Z$).

To simplify things, suppose the set $W$ consists of a single node $w$. Now pick the node $w$ and remove it from the graph $G$ for Bayesian network $B$. By “remove a node” we mean delete that node $w$ along with all edges to/from it; we leave the conditional probability tables for nodes in $X \cup Z$ as they are. Let us call the remaining graph (after removal of $w$) $G'$ and the resulting Bayesian network $B'$.

The node $w$ is called a barren node relative to $X \cup Z$ because it is irrelevant to probability computations concerning $X \cup Z$. Prove it is irrelevant by showing that $P(X|Z)$ in Bayesian network $B$ is equal to $P(X|Z)$ in Bayesian network $B'$.

[10 points]