1. (Exercise 4.1 in the book) Complete the analysis of example 4.4, showing that the distribution $P$ defined in the example does not factorize over $\mathcal{H}$. (Hint: Use a proof by contradiction). If you read Theorem 4.1 to Example 4.4 on pages 115-116, it will really help with this question. [10 points]

2. (Exercise 4.10 in the book) We define the following properties for a set of independencies:
   - **Strong Union:** $(X \perp Y | Z) \implies (X \perp Y | Z, W)$
     In other words, additional evidence $W$ cannot induce dependence
   - **Transitivity:** For all disjoint sets $X, Y, Z$ and all variables $A$:
     $$\neg (X \perp A | Z) \& \neg (A \perp Y | Z) \implies \neg (X \perp Y | Z)$$
     Intuitively, this statement asserts that if $X$ and $Y$ are both correlated with some $A$ (given $Z$), then they are also correlated with each other (given $Z$). We can also write the contrapositive of this statement, which is less obvious but easier to read.
     For all $X, Y, Z, A$:
     $$(X \perp Y | Z) \rightarrow (X \perp A | Z) \lor (A \perp Y | Z).$$

Prove that if $I = I(H)$ for some Markov network $H$, then $I$ satisfies strong union and transitivity. (If you aren’t familiar with some of the symbols, $\neg$ means logical NOT, $\&$ means logical AND and $\lor$ means logical OR) [10 points]

3. (Exercise 4.14 in the book). The Markov blanket of a node $X$ in a Bayesian network $G$, denoted $MB_G(X)$, is defined to be the nodes consisting of $X$’s parents, $X$’s children, and other parents of $X$’s children. Show the following:

   a) For any variable $X$, let $W = X - \{X\} - MB_G(X)$ where $X$ is the set of all random variables in the Bayesian network $G$. Then $d-sep_G(X; W | MB_G(X))$ [10 points]

   b) The set $MB_G(X)$ is the minimal set for which this property holds. [10 points]

4. (Exercise 4.18 in the book) Let $G$ be a Bayesian network structure and $\mathcal{H}$ a Markov network structure over $X$ such that the skeleton of $G$ is precisely $\mathcal{H}$. Prove that if $G$ has no immoralities, then $I(G) = I(\mathcal{H})$. [5 points]