1. Consider a stable LTI system whose transfer function is:

\[ H(z) = \left( \frac{1 - 2z^{-1}}{1 - 0.5e^{\frac{\pi}{2}j}z^{-1}} \right) \left( \frac{1 + z^{-1}}{1 - 0.5e^{-\frac{\pi}{2}j}z^{-1}} \right) \]

a. Write the difference equation that is satisfied by the input and the output of the system.
b. Plot the pole-zero diagram and indicate the ROC for the system function.
c. Find and sketch \(|H(e^{jw})|\)
d. State whether the following are true or false about the system:
   I. The system is causal.
   II. The magnitude of the frequency response has a peak at approximately \(w = \pm \frac{\pi}{2}\)
   III. The inverse system can be stable and causal.

2. Consider a stable LTI system whose transfer function is:

\[ H(z) = \left( \frac{1 - 0.9e^{\frac{\pi}{4}j}z^{-1}}{1 - 0.99e^{\frac{\pi}{2}j}z^{-1}} \right) \left( \frac{1 - 0.9e^{-\frac{\pi}{4}j}z^{-1}}{1 - 0.99e^{-\frac{\pi}{2}j}z^{-1}} \right) \]

a. Find and sketch \(|H(e^{jw})|\)
b. Find and sketch the group delay of the transfer function in the frequency domain.

3. In this problem, we demonstrate that, for a rational z-transform, a factor of the form \(z - z_0\) and a factor of the form \(z^{-1}z_0\) contribute the same phase.

(a) Let \(H(z) = z - \frac{1}{a}\), where \(a\) is real and \(0 < a < 1\). Sketch the poles and zeros of the system, including an indication of those at \(z = \infty\). Determine \(\angle H(e^{jw})\), the phase of the system.

(b) Let \(G(z) = \frac{1}{1-az^{-1}}\). Sketch the pole-zero diagram of \(G(z)\). Determine \(\angle G(e^{jw})\), the phase of the system, and show that it is identical to \(\angle H(e^{jw})\).
4. **Fig. 1.** shows the pole-zero plots for four different LTI systems. Based on these plots, state whether each system is an all-pass system.

![Pole-Zero plots of 4 different LTI systems (for Prob. 4)](image)

- (a) $H_1(z)$
- (b) $H_2(z)$
- (c) $H_3(z)$
- (d) $H_4(z)$

**Fig. 1.** Pole-Zero plots of 4 different LTI systems (for Prob. 4)

5. Consider the all-pass system described by the following $z$-domain transfer function:

$$H(z) = \left( \frac{0.5 + 0.5j + z^{-1}}{1 + (0.5 - 0.5j)z^{-1}} \right) \left( \frac{0.5 - 0.5j + z^{-1}}{1 + (0.5 + 0.5j)z^{-1}} \right)$$

- (a) Sketch the zeros and poles of this system.
- (b) Write and sketch the amplitude of the transfer function in the frequency domain.
- (c) Write and sketch the group delay of the transfer function in the frequency domain.